Solution of ECE 315 Test 4 Su07

1. Find the numerical values of the constants.

(a) $7\,\mathrm{sinc}^2\left(2t-3\right) \overset{\mathrm{F}}{\longleftrightarrow} A\,\mathrm{tri}\left(af\right)e^{jbf}$

A = _____ , a = _____ , b = _____

$7\,\mathrm{sinc}^2\left(2t-3\right) = 7\,\mathrm{sinc}^2\left(2\left(t-3/2\right)\right) \overset{\mathrm{F}}{\longleftrightarrow} \left(7/2\right)\mathrm{tri}\left(f/2\right)e^{-j3\pi f}$

A = 7/2 , a = 1/2 , b = -3π

(b) $A + B\cos\left(b\left(t-c\right)\right) \overset{\mathrm{F}}{\longleftrightarrow} \left[5\delta\left(f\right) + 3\delta\left(f-4\right) + 3\delta\left(f+4\right)\right]e^{j\pi f/5}$

A = _____ , B = _____ , b = _____ , c = _____

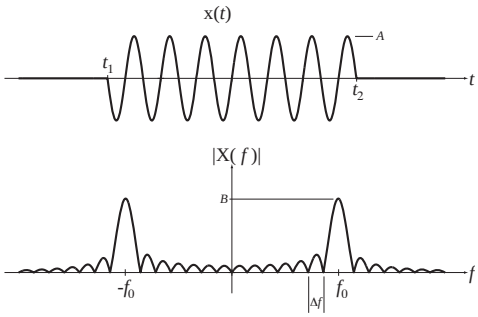
$A + B\cos\left(b\left(t-c\right)\right) \overset{\mathrm{F}}{\longleftrightarrow} 5\delta\left(f\right)e^{j\pi f/5} + 3\left[\delta\left(f-4\right) + \delta\left(f+4\right)\right]e^{j\pi f/5}$

$A + B\cos\left(b\left(t-c\right)\right) \overset{\mathrm{F}}{\longleftrightarrow} 5\delta\left(f\right) + 3\left[\delta\left(f-4\right) + \delta\left(f+4\right)\right]e^{j\pi f/5}$

$5 + 6\cos\left(8\pi\left(t+1/10\right)\right) \overset{\mathrm{F}}{\longleftrightarrow} 5\delta\left(f\right) + 3\left[\delta\left(f-4\right) + \delta\left(f+4\right)\right]e^{j\pi f/5}$

A = 5 , B = 6 , b = 8π , c = -1/10

2. In the graph below, let $t_1 = 0$, $t_2 = 2$, $A = 4$.



(Bottom graph for parameter identification only, not necessarily to scale.)

(a) Write an expression with numerical parameters for $x(t)$.

$$x(t) = -4\sin(7\pi t)\,\mathrm{rect}\left((t-1)/2\right)$$

(b) Find the numerical values for *B*, f_0 and Δf .

$$X(f) = -j2\left[\delta(f+7/2) - \delta(f-7/2)\right] * 2\,\mathrm{sinc}(2f)\,e^{-j2\pi f}$$

$$X(f) = -j4\left[\mathrm{sinc}\left(2(f+7/2)\right)e^{-j2\pi(f+7/2)} - \mathrm{sinc}\left(2(f-7/2)\right)e^{-j2\pi(f-7/2)}\right]$$

$B =$ <u>4</u> , $f_0 =$ <u>7/2</u> , $\Delta f =$ <u>1/2</u>

3. If $y(t) = x(t) * h(t)$, $y(t) \xleftrightarrow{F} Y(f)$, $x(t) = \text{sinc}(4t)$ and $h(t) = \text{rect}(t-1)$

what is the numerical value of $Y(1/2)$?

$$Y(f) = (1/4)\,\text{rect}(f/4)\,\text{sinc}(f)\,e^{-j2\pi f}$$

$$Y(1/2) = (1/4)\,\text{rect}(1/8)\,\text{sinc}(1/2)\,e^{-j\pi} = -(1/4)\frac{\sin(\pi/2)}{\pi/2} = -1/2\pi = -0.1592$$

4. If the *f* form of a Fourier transform is $X(f) = 9\left[\delta(f-4) + \delta(f+4)\right]$ and the ω form can be written as

$$X(j\omega) = A\left[\delta(\omega - b) + \delta(\omega + b)\right]$$

find the numerical values of *A* and *b* $(A \neq 9)$.

$$X(j\omega) = 9\left[\delta(\omega/2\pi - 4) + \delta(\omega/2\pi + 4)\right]$$

Then, using the scaling property of the impulse,

$$X(j\omega) = 18\pi\left[\delta(\omega - 8\pi) + \delta(\omega + 8\pi)\right].$$

A = $\underline{18\pi}$, *b* = $\underline{8\pi}$

Solution of ECE 315 Test 4 Su07

1. Find the numerical values of the constants.

(a) $13\text{sinc}^2\left(3t-2\right) \overset{\text{F}}{\longleftrightarrow} A\,\text{tri}\left(af\right)e^{jbf}$

A = _____ , a = _____ , b = _____

$13\text{sinc}^2\left(3t-2\right) = 13\text{sinc}^2\left(3\left(t-2/3\right)\right) \overset{\text{F}}{\longleftrightarrow} \left(13/3\right)\text{tri}\left(f/3\right)e^{-j4\pi f/3}$

A = 13/3 , a = 1/3 , b = $-4\pi/3$

(b) $A + B\cos\left(b\left(t-c\right)\right) \overset{\text{F}}{\longleftrightarrow} \left[8\delta\left(f\right) + 2\delta\left(f-4\right) + 2\delta\left(f+4\right)\right]e^{j\pi f/10}$

A = _____ , B = _____ , b = _____ , c = _____

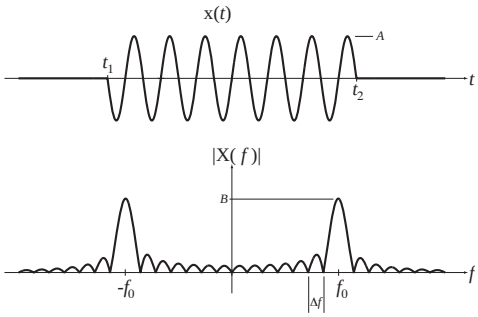
$A + B\cos\left(b\left(t-c\right)\right) \overset{\text{F}}{\longleftrightarrow} 8\delta\left(f\right)e^{j\pi f/10} + 2\left[\delta\left(f-7\right) + \delta\left(f+7\right)\right]e^{j\pi f/10}$

$A + B\cos\left(b\left(t-c\right)\right) \overset{\text{F}}{\longleftrightarrow} 8\delta\left(f\right) + 2\left[\delta\left(f-7\right) + \delta\left(f+7\right)\right]e^{j\pi f/10}$

$8 + 4\cos\left(14\pi\left(t+1/20\right)\right) \overset{\text{F}}{\longleftrightarrow} 8\delta\left(f\right) + 2\left[\delta\left(f-7\right) + \delta\left(f+7\right)\right]e^{j\pi f/10}$

A = 8 , B = 4 , b = 14π , c = -1/20

2. In the graph below, let $t_1 = 0$, $t_2 = 3$, $A = 9$.



(Bottom graph for parameter identification only, not necessarily to scale.)

(a) Write an expression with numerical parameters for $x(t)$.

$$x(t) = -9\sin\left((14/3)\pi t\right)\text{rect}\left((t-3/2)/3\right)$$

(b) Find the numerical values for *B*, f_0 and Δf .

$$X(f) = -j(9/2)\left[\delta(f+7/3) - \delta(f-7/3)\right] * 3\text{sinc}(3f)e^{-j3\pi f}$$

$$X(f) = -j(27/2)\left[\text{sinc}\left(3(f+7/3)\right)e^{-j3\pi(f+7/3)} - \text{sinc}\left(3(f-7/3)\right)e^{-j3\pi(f-7/3)}\right]$$

B = 27/2 , f_0 = 7/3 , Δf = 1/3

3. If $y(t) = x(t) * h(t)$, $y(t) \xleftrightarrow{F} Y(f)$, $x(t) = \text{sinc}(5t)$ and $h(t) = \text{rect}(t - 1/2)$

what is the numerical value of $Y(1/2)$?

$$Y(f) = (1/5)\,\text{rect}(f/5)\,\text{sinc}(f)\,e^{-j\pi f}$$

$$Y(1/2) = (1/5)\,\text{rect}(1/10)\,\text{sinc}(1/2)\,e^{-j\pi/2} = -(j/5)\frac{\sin(\pi/2)}{\pi/2} = -j2/5\pi = -0.1273$$

4. If the *f* form of a Fourier transform is $X(f) = 20[\delta(f - 35) + \delta(f + 35)]$ and the ω form can be written as

$$X(j\omega) = A[\delta(\omega - b) + \delta(\omega + b)]$$

find the numerical values of *A* and *b* $(A \neq 20)$.

$$X(j\omega) = 20[\delta(\omega/2\pi - 35) + \delta(\omega/2\pi + 35)]$$

Then, using the scaling property of the impulse,

$$X(j\omega) = 40\pi[\delta(\omega - 70\pi) + \delta(\omega + 70\pi)]\ .$$

A = $\underline{40\pi}$, *b* = $\underline{70\pi}$