Solution of 315 Test 11 F07

1. Let $\mathrm{x}[n] = \delta[n+2] - \delta[n-2]$.

   (a) Using the definition of the DTFT $\mathrm{X}(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} \mathrm{x}[n] e^{-j\Omega n}$ find $\mathrm{X}(e^{j\Omega})$.

$$\mathrm{X}(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} (\delta[n+2] - \delta[n-2]) e^{-j\Omega n} = e^{j2\Omega} - e^{-j2\Omega} = j2\sin(2\Omega)$$

   (b) At what numerical value(s) of Ω in the range $-\pi \le \Omega < \pi$ is $|\mathrm{X}(e^{j\Omega})|$ zero?

$$\sin(2\Omega) = 0 \Rightarrow 2\Omega = n\pi \Rightarrow \Omega = n\pi / 2 \ \ , \ n \text{ any integer}$$

$$\Omega = -\pi, -\pi/2, 0, \pi/2$$

2. If $\mathrm{X}(F) = \delta_1(F - 1/10) + \delta_1(F + 1/10) + \delta_{1/16}(F)$ and $\mathrm{x}[n] \xleftrightarrow{\mathcal{F}} \mathrm{X}(F)$, what is the fundamental period of $\mathrm{x}[n]$?

$$\delta_{N_0}[n] \xleftrightarrow{\mathcal{F}} (1/N_0)\delta_{1/N_0}(F)$$

$$\delta_{16}[n] \xleftrightarrow{\mathcal{F}} (1/16)\delta_{1/16}(F)$$

$$16\delta_{16}[n] \xleftrightarrow{\mathcal{F}} \delta_{1/16}(F)$$

$$\cos(2\pi F_0 n) \xleftrightarrow{\mathcal{F}} (1/2)[\delta_1(F - F_0) + \delta_1(F + F_0)]$$

$$2\cos(2\pi n/10) \xleftrightarrow{\mathcal{F}} \delta_1(F - 1/10) + \delta_1(F + 1/10)$$

$$16\delta_{16}[n] + 2\cos(2\pi n/10) \xleftrightarrow{\mathcal{F}} \delta_1(F - 1/10) + \delta_1(F + 1/10) + \delta_{1/16}(F)$$

Fundamental period is the least common multiple of 10 and 16 which is 80.

Solution of 315 Test 11 F07

1. Let $\mathrm{x}[n] = \delta[n+2] + \delta[n-2]$.

   (a) Using the definition of the DTFT $\mathrm{X}(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} \mathrm{x}[n] e^{-j\Omega n}$ find $\mathrm{X}(e^{j\Omega})$.

$$\mathrm{X}(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} (\delta[n+2] + \delta[n-2]) e^{-j\Omega n} = e^{j2\Omega} + e^{-j2\Omega} = 2\cos(2\Omega)$$

   (b) At what numerical value(s) of Ω in the range $-\pi \le \Omega < \pi$ is $|\mathrm{X}(e^{j\Omega})|$ zero?

$$\cos(2\Omega) = 0 \Rightarrow 2\Omega = n\pi + \pi/2 \Rightarrow \Omega = n\pi/2 + \pi/4 \ , \ n \text{ any integer}$$

$$\Omega = -3\pi/4, -\pi/4, \pi/4, 3\pi/4$$

2. If $\mathrm{X}(F) = \delta_1(F - 1/12) + \delta_1(F + 1/12) + \delta_{1/16}(F)$ and $\mathrm{x}[n] \stackrel{\mathcal{F}}{\longleftrightarrow} \mathrm{X}(F)$, what is the fundamental period of $\mathrm{x}[n]$?

$$\delta_{N_0}[n] \stackrel{\mathcal{F}}{\longleftrightarrow} (1/N_0)\delta_{1/N_0}(F)$$

$$\delta_{16}[n] \stackrel{\mathcal{F}}{\longleftrightarrow} (1/16)\delta_{1/16}(F)$$

$$16\delta_{16}[n] \stackrel{\mathcal{F}}{\longleftrightarrow} \delta_{1/16}(F)$$

$$\cos(2\pi F_0 n) \stackrel{\mathcal{F}}{\longleftrightarrow} (1/2)[\delta_1(F - F_0) + \delta_1(F + F_0)]$$

$$2\cos(2\pi n/10) \stackrel{\mathcal{F}}{\longleftrightarrow} \delta_1(F - 1/12) + \delta_1(F + 1/12)$$

$$16\delta_{16}[n] + 2\cos(2\pi n/12) \stackrel{\mathcal{F}}{\longleftrightarrow} \delta_1(F - 1/10) + \delta_1(F + 1/10) + \delta_{1/16}(F)$$

Fundamental period is the least common multiple of 12 and 16 which is 48.

1. Let $x[n] = \delta[n+2] - \delta[n-2]$.

(a) Using the definition of the DTFT $X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\Omega n}$ find $X(e^{j\Omega})$.

$$X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} (\delta[n+2] - \delta[n-2]) e^{-j\Omega n} = e^{j2\Omega} - e^{-j2\Omega} = j2\sin(2\Omega)$$

(b) At what numerical value(s) of Ω in the range $-\pi \le \Omega < \pi$ is $|X(e^{j\Omega})|$ zero?

$$\sin(2\Omega) = 0 \Rightarrow 2\Omega = n\pi \Rightarrow \Omega = n\pi / 2 \ \ , \ n \text{ any integer}$$

$$\Omega = -\pi, -\pi/2, 0, \pi/2$$

2. If $X(F) = \delta_1(F - 1/10) + \delta_1(F + 1/10) + \delta_{1/12}(F)$ and $x[n] \xleftrightarrow{\mathcal{F}} X(F)$, what is the fundamental period of $x[n]$?

$$\delta_{N_0}[n] \xleftrightarrow{\mathcal{F}} (1/N_0)\delta_{1/N_0}(F)$$

$$\delta_{12}[n] \xleftrightarrow{\mathcal{F}} (1/12)\delta_{1/12}(F)$$

$$12\delta_{12}[n] \xleftrightarrow{\mathcal{F}} \delta_{1/12}(F)$$

$$\cos(2\pi F_0 n) \xleftrightarrow{\mathcal{F}} (1/2)[\delta_1(F - F_0) + \delta_1(F + F_0)]$$

$$2\cos(2\pi n/10) \xleftrightarrow{\mathcal{F}} \delta_1(F - 1/10) + \delta_1(F + 1/10)$$

$$12\delta_{12}[n] + 2\cos(2\pi n/10) \xleftrightarrow{\mathcal{F}} \delta_1(F - 1/10) + \delta_1(F + 1/10) + \delta_{1/16}(F)$$

Fundamental period is the least common multiple of 10 and 12 which is 60.