Solution of 315 Test 12 F07

1. If $\mathrm{x}[n] = n^2(\mathrm{u}[n] - \mathrm{u}[n-3])$ and $\mathrm{x}[n] \xleftrightarrow{\mathcal{F}} \mathrm{X}(F)$, what is the numerical value of $\mathrm{X}(0)$?
$\mathrm{X}(0) =$ ____________________

$$\mathrm{X}(F) = \sum_{n=-\infty}^{\infty} \mathrm{x}[n] e^{-j2\pi Fn} \Rightarrow \mathrm{X}(0) = \sum_{n=-\infty}^{\infty} \mathrm{x}[n] = \sum_{n=-\infty}^{\infty} n^2(\mathrm{u}[n] - \mathrm{u}[n-3])$$

$$\mathrm{X}(0) = \sum_{n=0}^{2} n^2 = 0 + 1 + 4 = 5$$

2. If $\mathrm{X}(F) = 3[\delta_1(F - 1/4) + \delta_1(F + 1/4)] - j4[\delta_1(F + 1/9) - \delta(F - 1/9)]$ and $\mathrm{x}[n] \xleftrightarrow{\mathcal{F}} \mathrm{X}(F)$, what is the numerical fundamental period of $\mathrm{x}[n]$? $N_0 =$ ____________________

$$\mathrm{x}[n] = 6\cos(2\pi n/4) - 8\sin(2\pi n/9)$$

The fundamental period of $\mathrm{x}[n]$ is the least common multiple of the fundamental periods of the sine and cosine which are added to form it. The least common multiple of 4 and 9 is 36.

3. If $\mathrm{x}[n] = \mathrm{tri}(n/3)$ and $\mathrm{x}_p[n] = \mathrm{x}[n] * \delta_{10}[n]$ and $\mathrm{x}_p[n] \xleftrightarrow{\mathcal{FS}} \mathrm{X}_p[k]$, what is the numerical value of $\mathrm{X}_p[0]$? $\mathrm{X}_p[0] =$ ____________________

$\mathrm{X}_p[0]$ is the average value of $\mathrm{x}_p[n]$. The average value of any periodic signal is the sum of the impulse strengths in one period, divided by the period. In this case

$$\mathrm{X}_p[0] = \frac{1}{10}\sum_{n=-5}^{4} \mathrm{tri}(n/3) = \frac{1}{10}\sum_{n=-3}^{3} \mathrm{tri}(n/3) = \frac{0 + 1/3 + 2/3 + 1 + 2/3 + 1/3 + 0}{10} = 0.3$$

Solution of 315 Test 12 F07

1. If $\mathrm{x}[n] = n^2(\mathrm{u}[n] - \mathrm{u}[n-4])$ and $\mathrm{x}[n] \xleftrightarrow{\mathcal{F}} \mathrm{X}(F)$, what is the numerical value of $\mathrm{X}(0)$?
$\mathrm{X}(0) =$ ____________________

$$\mathrm{X}(F) = \sum_{n=-\infty}^{\infty} \mathrm{x}[n] e^{-j2\pi Fn} \Rightarrow \mathrm{X}(0) = \sum_{n=-\infty}^{\infty} \mathrm{x}[n] = \sum_{n=-\infty}^{\infty} n^2(\mathrm{u}[n] - \mathrm{u}[n-3])$$

$$\mathrm{X}(0) = \sum_{n=0}^{3} n^2 = 0 + 1 + 4 + 9 = 14$$

2. If $\mathrm{X}(F) = 3[\delta_1(F-1/7) + \delta_1(F+1/7)] - j4[\delta_1(F+1/9) - \delta(F-1/9)]$ and $\mathrm{x}[n] \xleftrightarrow{\mathcal{F}} \mathrm{X}(F)$, what is the numerical fundamental period of $\mathrm{x}[n]$? $N_0 =$ ____________________

$$\mathrm{x}[n] = 6\cos(2\pi n/7) - 8\sin(2\pi n/9)$$

The fundamental period of $\mathrm{x}[n]$ is the least common multiple of the fundamental periods of the sine and cosine which are added to form it. The least common multiple of 7 and 9 is 63.

3. If $\mathrm{x}[n] = \mathrm{tri}(n/3)$ and $\mathrm{x}_p[n] = \mathrm{x}[n] * \delta_8[n]$ and $\mathrm{x}_p[n] \xleftrightarrow{\mathcal{FS}} \mathrm{X}_p[k]$, what is the numerical value of $\mathrm{X}_p[0]$? $\mathrm{X}_p[0] =$ ________________________

$\mathrm{X}_p[0]$ is the average value of $\mathrm{x}_p[n]$. The average value of any periodic signal is the sum of the impulse strengths in one period, divided by the period. In this case

$$\mathrm{X}_p[0] = \frac{1}{8}\sum_{n=-5}^{4} \mathrm{tri}(n/3) = \frac{1}{8}\sum_{n=-3}^{3} \mathrm{tri}(n/3) = \frac{0 + 1/3 + 2/3 + 1 + 2/3 + 1/3 + 0}{8} = 3/8 = 0.375$$

Solution of 315 Test 12 F07

1. If $\mathrm{x}[n] = n^2(\mathrm{u}[n] - \mathrm{u}[n-5])$ and $\mathrm{x}[n] \xleftrightarrow{\mathcal{F}} \mathrm{X}(F)$, what is the numerical value of $\mathrm{X}(0)$?
$\mathrm{X}(0) =$ ____________________

$$\mathrm{X}(F) = \sum_{n=-\infty}^{\infty} \mathrm{x}[n] e^{-j2\pi Fn} \Rightarrow \mathrm{X}(0) = \sum_{n=-\infty}^{\infty} \mathrm{x}[n] = \sum_{n=-\infty}^{\infty} n^2(\mathrm{u}[n] - \mathrm{u}[n-5])$$

$$\mathrm{X}(0) = \sum_{n=0}^{4} n^2 = 0 + 1 + 4 + 9 + 16 = 30$$

2. If $\mathrm{X}(F) = 3[\delta_1(F - 1/4) + \delta_1(F + 1/4)] - j4[\delta_1(F + 1/7) - \delta(F - 1/7)]$ and $\mathrm{x}[n] \xleftrightarrow{\mathcal{F}} \mathrm{X}(F)$, what is the numerical fundamental period of $\mathrm{x}[n]$? $N_0 =$ ____________________

$$\mathrm{x}[n] = 6\cos(2\pi n/4) - 8\sin(2\pi n/7)$$

The fundamental period of $\mathrm{x}[n]$ is the least common multiple of the fundamental periods of the sine and cosine which are added to form it. The least common multiple of 4 and 7 is 28.

3. If $\mathrm{x}[n] = \mathrm{tri}(n/3)$ and $\mathrm{x}_p[n] = \mathrm{x}[n] * \delta_{12}[n]$ and $\mathrm{x}_p[n] \xleftrightarrow{\mathcal{FS}} \mathrm{X}_p[k]$, what is the numerical value of $\mathrm{X}_p[0]$? $\mathrm{X}_p[0] =$ ______________________

$\mathrm{X}_p[0]$ is the average value of $\mathrm{x}_p[n]$. The average value of any periodic signal is the sum of the impulse strengths in one period, divided by the period. In this case

$$\mathrm{X}_p[0] = \frac{1}{12}\sum_{n=-5}^{4} \mathrm{tri}(n/3) = \frac{1}{12}\sum_{n=-3}^{3} \mathrm{tri}(n/3) = \frac{0 + 1/3 + 2/3 + 1 + 2/3 + 1/3 + 0}{12} = 0.25$$