Solution of ECE 315 Test 11 F08

1. $$7\,\text{rect}_3[n] * 4\sin(2\pi n/12) \xleftrightarrow{\mathcal{F}} A\,\text{drcl}(F,a)[\delta_1(F+b)-\delta_1(F-b)]$$

$$\text{rect}_3[n] \xleftrightarrow{\mathcal{F}} 7\,\text{drcl}(F,7)$$

$$\sin(2\pi n/12) \xleftrightarrow{\mathcal{F}} (j/2)[\delta_1(F+1/12)-\delta_1(F-1/12)]$$

$$7\,\text{rect}_3[n] * 4\sin(2\pi n/12) \xleftrightarrow{\mathcal{F}} j98\,\text{drcl}(F,7)[\delta_1(F+1/12)-\delta_1(F-1/12)]$$

2. $$j42\,\text{drcl}(F,5)[\delta_1(F+1/16)-\delta_1(F-1/16)]e^{j4\pi F} = A\left\{\begin{matrix}\delta_1(F+1/16)-\delta_1(F-1/16) \\ -j[\delta_1(F+1/16)+\delta_1(F-1/16)]\end{matrix}\right\}$$

$$j42\,\text{drcl}(F,5)[\delta_1(F+1/16)-\delta_1(F-1/16)]e^{j4\pi F} = j42\begin{bmatrix}\text{drcl}(-1/16,5)e^{-j\pi/4}\delta_1(F+1/16) \\ -\text{drcl}(1/16,5)e^{j\pi/4}\delta_1(F-1/16)\end{bmatrix}$$

$$j42\,\text{drcl}(F,5)[\delta_1(F+1/16)-\delta_1(F-1/16)]e^{j4\pi F} = j42\,\text{drcl}(1/16,5)\begin{bmatrix}\dfrac{1-j}{\sqrt{2}}\delta_1(F+1/16) \\ -\dfrac{1+j}{\sqrt{2}}\delta_1(F-1/16)\end{bmatrix}$$

$$j42\,\text{drcl}(F,5)[\delta_1(F+1/16)-\delta_1(F-1/16)]e^{j4\pi F} = j\frac{35.8006}{\sqrt{2}}\begin{bmatrix}\delta_1(F+1/16)-\delta_1(F-1/16) \\ -j\delta_1(F+1/16)-j\delta_1(F-1/16)\end{bmatrix}$$

$$j42\,\text{drcl}(F,5)[\delta_1(F+1/16)-\delta_1(F-1/16)]e^{j4\pi F} = j25.3148\left\{\begin{matrix}\delta_1(F+1/16)-\delta_1(F-1/16) \\ -j[\delta_1(F+1/16)+\delta_1(F-1/16)]\end{matrix}\right\}$$

3. $$A\cos\left(\frac{2\pi(n-n_0)}{N_0}\right) \xleftrightarrow{\mathcal{F}} j\frac{36}{\sqrt{2}}\{(1-j)\delta_1(F+1/16)-(1+j)\delta_1(F-1/16)\}$$

$$A\cos\left(\frac{2\pi(n-n_0)}{N_0}\right) \xleftrightarrow{\mathcal{F}} (A/2)[\delta_1(F-1/N_0)+\delta_1(F+1/N_0)]e^{-j2\pi F n_0}$$

$$A\cos\left(\frac{2\pi(n-n_0)}{N_0}\right) \xleftrightarrow{\mathcal{F}} (A/2)[e^{-j2\pi n_0/N_0}\delta_1(F-1/N_0)+e^{j2\pi n_0/N_0}\delta_1(F+1/N_0)]$$

$$N_0 = 16$$

$$j\frac{36}{\sqrt{2}}\{(1-j)\delta_1(F+1/16)-(1+j)\delta_1(F-1/16)\} = (A/2)[e^{-j2\pi n_0/16}\delta_1(F-1/16)+e^{j2\pi n_0/16}\delta_1(F+1/16)]$$

$$j\frac{36}{\sqrt{2}}(1-j) = (A/2)e^{j2\pi n_0/16} \quad , \; -j\frac{36}{\sqrt{2}}(1+j) = (A/2)e^{-j2\pi n_0/16}$$

Dividing the first equation by the second,

$$-\frac{1-j}{1+j} = \frac{e^{j2\pi n_0/16}}{e^{-j2\pi n_0/16}} \Rightarrow -\frac{1-j}{1+j} = e^{j4\pi n_0/16}$$

$$-\frac{e^{-j\pi/4}}{e^{j\pi/4}} = e^{j4\pi n_0/16} \Rightarrow -e^{-j\pi/2} = e^{j4\pi n_0/16} \Rightarrow e^{j\pi/2} = e^{j4\pi n_0/16} \Rightarrow n_0 = 2$$

$$j\frac{36}{\sqrt{2}}(1-j) = (A/2)e^{j\pi/4} = (A/2)\frac{1+j}{\sqrt{2}} \Rightarrow (A/2) = j36\frac{1-j}{1+j} = j36\frac{e^{-j\pi/4}}{e^{j\pi/4}} = j36\underbrace{e^{-j\pi/2}}_{=-j} = 36 \Rightarrow A = 72$$

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1. $$7\,\text{rect}_2[n] * 4\sin(2\pi n/8) \xleftrightarrow{\mathcal{F}} A\,\text{drcl}(F,a)\left[\delta_1(F+b)-\delta_1(F-b)\right]$$

$$\text{rect}_2[n] \xleftrightarrow{\mathcal{F}} 5\,\text{drcl}(F,5)$$

$$\sin(2\pi n/8) \xleftrightarrow{\mathcal{F}} (j/2)\left[\delta_1(F+1/8)-\delta_1(F-1/8)\right]$$

$$7\,\text{rect}_2[n] * 4\sin(2\pi n/8) \xleftrightarrow{\mathcal{F}} j70\,\text{drcl}(F,5)\left[\delta_1(F+1/8)-\delta_1(F-1/8)\right]$$

2. $$j24\,\text{drcl}(F,7)\left[\delta_1(F+1/12)-\delta_1(F-1/12)\right]e^{j3\pi F} = A\left\{\begin{array}{l}\delta_1(F+1/12)-\delta_1(F-1/12)\\ -j\left[\delta_1(F+1/12)+\delta_1(F-1/12)\right]\end{array}\right\}$$

$$j24\,\text{drcl}(F,7)\left[\delta_1(F+1/12)-\delta_1(F-1/12)\right]e^{j3\pi F} = j24\left[\begin{array}{l}\text{drcl}(-1/12,7)e^{-j\pi/4}\delta_1(F+1/12)\\ -\text{drcl}(1/12,7)e^{j\pi/4}\delta_1(F-1/12)\end{array}\right]$$

$$j24\,\text{drcl}(F,7)\left[\delta_1(F+1/12)-\delta_1(F-1/12)\right]e^{j3\pi F} = j24\,\text{drcl}(1/12,7)\left[\begin{array}{l}\frac{1-j}{\sqrt{2}}\delta_1(F+1/12)\\ -\frac{1+j}{\sqrt{2}}\delta_1(F-1/12)\end{array}\right]$$

$$j24\,\text{drcl}(F,7)\left[\delta_1(F+1/12)-\delta_1(F-1/12)\right]e^{j3\pi F} = j\frac{12.7956}{\sqrt{2}}\left[\begin{array}{l}\delta_1(F+1/12)-\delta_1(F-1/12)\\ -j\delta_1(F+1/12)-j\delta_1(F-1/12)\end{array}\right]$$

$$j24\,\text{drcl}(F,7)\left[\delta_1(F+1/12)-\delta_1(F-1/12)\right]e^{j3\pi F} = j9.0479\left\{\begin{array}{l}\delta_1(F+1/12)-\delta_1(F-1/12)\\ -j\left[\delta_1(F+1/12)+\delta_1(F-1/12)\right]\end{array}\right\}$$

3. $$A\cos\left(\frac{2\pi(n-n_0)}{N_0}\right) \xleftrightarrow{\mathcal{F}} j\frac{12}{\sqrt{2}}\left\{(1-j)\delta_1(F+1/8)-(1+j)\delta_1(F-1/8)\right\}$$

$$A\cos\left(\frac{2\pi(n-n_0)}{N_0}\right) \xleftrightarrow{\mathcal{F}} (A/2)\left[\delta_1(F-1/N_0)+\delta_1(F+1/N_0)\right]e^{-j2\pi F n_0}$$

$$A\cos\left(\frac{2\pi(n-n_0)}{N_0}\right) \xleftrightarrow{\mathcal{F}} (A/2)\left[e^{-j2\pi n_0/N_0}\delta_1(F-1/N_0)+e^{j2\pi n_0/N_0}\delta_1(F+1/N_0)\right]$$

$$N_0 = 8$$

$$j\frac{12}{\sqrt{2}}\left\{(1-j)\delta_1(F+1/8)-(1+j)\delta_1(F-1/8)\right\} = (A/2)\left[e^{-j2\pi n_0/8}\delta_1(F-1/8)+e^{j2\pi n_0/8}\delta_1(F+1/8)\right]$$

$$j\frac{12}{\sqrt{2}}(1-j) = (A/2)e^{j2\pi n_0/8} \quad , \quad -j\frac{12}{\sqrt{2}}(1+j) = (A/2)e^{-j2\pi n_0/8}$$

Dividing the first equation by the second,

$$-\frac{1-j}{1+j} = \frac{e^{j2\pi n_0/8}}{e^{-j2\pi n_0/8}} \Rightarrow -\frac{1-j}{1+j} = e^{j4\pi n_0/8}$$

$$-\frac{e^{-j\pi/4}}{e^{j\pi/4}} = e^{j4\pi n_0/8} \Rightarrow -e^{-j\pi/2} = e^{j4\pi n_0/8} \Rightarrow e^{j\pi/2} = e^{j4\pi n_0/8} \Rightarrow n_0 = 1$$

$$j\frac{12}{\sqrt{2}}(1-j) = (A/2)e^{j\pi/4} = (A/2)\frac{1+j}{\sqrt{2}} \Rightarrow (A/2) = j12\frac{1-j}{1+j} = j12\frac{e^{-j\pi/4}}{e^{j\pi/4}} = j12\underbrace{e^{-j\pi/2}}_{=-j} = 12 \Rightarrow A = 24$$