Solution of EECS 315 Test 11 F13

1. If $\mathrm{x}(t) \xleftrightarrow{\mathcal{F}} \mathrm{X}(f) = \dfrac{5}{j2\pi f + 15}$ and $\mathrm{y}(t) = \mathrm{x}(3t)$ and $\mathrm{y}(t) \xleftrightarrow{\mathcal{F}} \mathrm{Y}(f)$, what are the numerical magnitude and phase (in radians) of $\mathrm{Y}(6)$?

   $|\mathrm{Y}(6)|$ = ____________ $\measuredangle \mathrm{Y}(6)$ = ____________ radians

$$\mathrm{Y}(f) = \frac{1}{3}\mathrm{X}\left(\frac{f}{3}\right) = \frac{1}{3}\frac{5}{j2\pi f/3 + 15} = \frac{5}{j2\pi f + 45} \Rightarrow \mathrm{Y}(6) = \frac{5}{j12\pi + 45} = 0.0852\measuredangle -0.697 \text{ radians}$$

2. If $\mathrm{x}(t) = \delta_{T_0}(t) * \mathrm{rect}(t/4)$ and $\mathrm{x}(t) \xleftrightarrow{\mathcal{F}} \mathrm{X}(f)$ find three different numerical values of T_0 for which $\mathrm{X}(f) = A\delta(f)$ and the corresponding numerical impulse strengths A.

   T_0 = ____________ A = ____________

   T_0 = ____________ A = ____________

   T_0 = ____________ A = ____________

$$\mathrm{x}(t) = \delta_{T_0}(t) * \mathrm{rect}(t/4) \xleftrightarrow{\mathcal{F}} \mathrm{X}(f) = (1/T_0)\delta_{1/T_0}(f) 4\,\mathrm{sinc}(4f)$$

If T_0 is chosen to make the impulses in $\mathrm{X}(f)$ all fall at integer values of $4f$ then all the impulses in the periodic impulse, except the one at $4f = 0$, will have zero strength. The integer values of $4f$ are integer multiples of 1/4. So $1/T_0$ should be an integer multiple of 1/4. In general $1/T_0 = k/4$, k an integer , implying that $T_0 = 4/k$, k an integer , with a corresponding impulse strength of k. The simplest answers are

$T_0 = 4$ $A = 1$

$T_0 = 2$ $A = 2$

$T_0 = 1$ $A = 4$

but there are infinitely many other correct answers.

3. An LTI continuous-time system has a frequency response $\mathrm{H}(f)=\dfrac{1500}{j8f+1000}$. It is excited by a signal whose Fourier transform is $\mathrm{X}(f)=\sum_{k=-\infty}^{\infty}\mathrm{sinc}\left(\dfrac{k}{12}\right)\delta(f-120k)$. The Fourier transform of the system response is $\mathrm{Y}(f)$. Find the numerical magnitude and phase (in radians) of the strength of the impulse in $\mathrm{Y}(f)$ occurring at $f = 480$ Hz .

Impulse strength magnitude = ____________ Impulse strength phase = ____________ radians

$$\mathrm{Y}(f)=\mathrm{X}(f)\mathrm{H}(f)=\frac{1500}{j8f+1000}\sum_{k=-\infty}^{\infty}\mathrm{sinc}\left(\frac{k}{12}\right)\delta(f-120k)$$

$$\mathrm{Y}(480)=\frac{1500}{j8\times 480+1000}\mathrm{sinc}\left(\frac{4}{12}\right)\delta(f-480)$$

$$\text{Impulse Strength}=\frac{1500}{j3840+1000}\mathrm{sinc}(1/3)=0.3126\measuredangle-1.316\text{ radians}$$

Solution of EECS 315 Test 11 F13

1. If $\mathrm{x}(t) \xleftrightarrow{\mathcal{F}} \mathrm{X}(f) = \dfrac{13}{j2\pi f + 7}$ and $\mathrm{y}(t) = \mathrm{x}(3t)$ and $\mathrm{y}(t) \xleftrightarrow{\mathcal{F}} \mathrm{Y}(f)$, what are the numerical magnitude and phase (in radians) of $\mathrm{Y}(6)$?

   $|\mathrm{Y}(6)|$ = ____________ $\measuredangle \mathrm{Y}(6)$ = ____________ radians

   $$\mathrm{Y}(f) = \frac{1}{3}\mathrm{X}\left(\frac{f}{3}\right) = \frac{1}{3}\frac{13}{j2\pi f/3 + 7} = \frac{13}{j2\pi f + 21} \Rightarrow \mathrm{Y}(6) = \frac{13}{j12\pi + 21} = 0.30125\measuredangle -1.0626 \text{ radians}$$

2. If $\mathrm{x}(t) = \delta_{T_0}(t) * \mathrm{rect}(t/6)$ and $\mathrm{x}(t) \xleftrightarrow{\mathcal{F}} \mathrm{X}(f)$ find three different numerical values of T_0 for which $\mathrm{X}(f) = A\delta(f)$ and the corresponding numerical impulse strengths A.

   T_0 = ____________ A = ____________

   T_0 = ____________ A = ____________

   T_0 = ____________ A = ____________

   $$\mathrm{x}(t) = \delta_{T_0}(t) * \mathrm{rect}(t/6) \xleftrightarrow{\mathcal{F}} \mathrm{X}(f) = (1/T_0)\delta_{1/T_0}(f) 6\,\mathrm{sinc}(6f)$$

   If T_0 is chosen to make the impulses in $\mathrm{X}(f)$ all fall at integer values of $6f$ then all the impulses in the periodic impulse, except the one at $6f = 0$, will have zero strength. The integer values of $6f$ are integer multiples of 1/6. So $1/T_0$ should be an integer multiple of 1/6. In general $1/T_0 = k/6$, k an integer , implying that $T_0 = 6/k$, k an integer , with a corresponding impulse strength of k. The simplest answers are

   $T_0 = 6$ $A = 1$

   $T_0 = 3$ $A = 2$

   $T_0 = 3/2$ $A = 4$

   but there are infinitely many other correct answers.

3. An LTI continuous-time system has a frequency response $\mathrm{H}(f)=\dfrac{1500}{j8f+2000}$. It is excited by a signal whose Fourier transform is $\mathrm{X}(f)=\sum_{k=-\infty}^{\infty}\mathrm{sinc}\left(\dfrac{k}{12}\right)\delta(f-120k)$. The Fourier transform of the system response is $\mathrm{Y}(f)$. Find the numerical magnitude and phase (in radians) of the strength of the impulse in $\mathrm{Y}(f)$ occurring at $f = 480$ Hz .

Impulse strength magnitude = ____________ Impulse strength phase = ____________ radians

$$\mathrm{Y}(f)=\mathrm{X}(f)\mathrm{H}(f)=\frac{1500}{j8f+2000}\sum_{k=-\infty}^{\infty}\mathrm{sinc}\left(\frac{k}{12}\right)\delta(f-120k)$$

$$\mathrm{Y}(480)=\frac{1500}{j8\times 480+2000}\mathrm{sinc}\left(\frac{4}{12}\right)\delta(f-480)$$

$$\text{Impulse Strength}=\frac{1500}{j3840+2000}\mathrm{sinc}(1/3)=0.2865\measuredangle-1.0906\text{ radians}$$

Solution of EECS 315 Test 11 F13

1. If $\mathrm{x}(t) \xleftrightarrow{\mathcal{F}} \mathrm{X}(f) = \dfrac{82}{j2\pi f + 11}$ and $\mathrm{y}(t) = \mathrm{x}(3t)$ and $\mathrm{y}(t) \xleftrightarrow{\mathcal{F}} \mathrm{Y}(f)$, what are the numerical magnitude and phase (in radians) of $\mathrm{Y}(6)$?

$|\mathrm{Y}(6)|$ = ____________ $\measuredangle \mathrm{Y}(6)$ = ____________ radians

$$\mathrm{Y}(f) = \frac{1}{3}\mathrm{X}\left(\frac{f}{3}\right) = \frac{1}{3}\frac{82}{j2\pi f/3 + 11} = \frac{82}{j2\pi f + 33} \Rightarrow \mathrm{Y}(6) = \frac{82}{j12\pi + 33} = 1.6367\measuredangle -0.8518 \text{ radians}$$

2. If $\mathrm{x}(t) = \delta_{T_0}(t) * \mathrm{rect}(t/10)$ and $\mathrm{x}(t) \xleftrightarrow{\mathcal{F}} \mathrm{X}(f)$ find three different numerical values of T_0 for which $\mathrm{X}(f) = A\delta(f)$ and the corresponding numerical impulse strengths A.

T_0 = ____________ A = ____________

T_0 = ____________ A = ____________

T_0 = ____________ A = ____________

$$\mathrm{x}(t) = \delta_{T_0}(t) * \mathrm{rect}(t/10) \xleftrightarrow{\mathcal{F}} \mathrm{X}(f) = (1/T_0)\delta_{1/T_0}(f)10\,\mathrm{sinc}(10f)$$

If T_0 is chosen to make the impulses in $\mathrm{X}(f)$ all fall at integer values of $10f$ then all the impulses in the periodic impulse, except the one at $10f = 0$, will have zero strength. The integer values of $10f$ are integer multiples of 1/10. So $1/T_0$ should be an integer multiple of 1/10. In general, implying that $T_0 = 10/k$, k an integer, with a corresponding impulse strength of k. The simplest answers are

T_0 = 10 A = 1

T_0 = 5 A = 2

T_0 = 5/2 A = 4

but there are infinitely many other correct answers.

3. An LTI continuous-time system has a frequency response $\mathrm{H}(f)=\frac{1500}{j8f+3000}$. It is excited by a signal whose Fourier transform is $\mathrm{X}(f)=\sum_{k=-\infty}^{\infty}\mathrm{sinc}\left(\frac{k}{12}\right)\delta(f-120k)$. The Fourier transform of the system response is $\mathrm{Y}(f)$. Find the numerical magnitude and phase (in radians) of the strength of the impulse in $\mathrm{Y}(f)$ occurring at $f = 480$ Hz .

Impulse strength magnitude = ____________ Impulse strength phase = ____________ radians

$$\mathrm{Y}(f)=\mathrm{X}(f)\mathrm{H}(f)=\frac{1500}{j8f+3000}\sum_{k=-\infty}^{\infty}\mathrm{sinc}\left(\frac{k}{12}\right)\delta(f-120k)$$

$$\mathrm{Y}(480)=\frac{1500}{j8\times 480+3000}\mathrm{sinc}\left(\frac{4}{12}\right)\delta(f-480)$$

$$\text{Impulse Strength}=\frac{1500}{j3840+3000}\mathrm{sinc}(1/3)=0.2546\measuredangle-0.9076\text{ radians}$$