Solution of EECS 315 Test 12 F13

1. Given the DTFT pair $x[n] \xleftrightarrow{\mathcal{F}} \frac{10}{1-0.6e^{-j2\pi F}}$ and $y[n] = \begin{cases} x[n/2] & , n/2 \text{ an integer} \\ 0 & , \text{otherwise} \end{cases}$, find the numerical magnitude and phase (in radians) of $Y(F)_{F=1/8}$.

$\left|Y(F)_{F=1/8}\right|$ = _____ , $\measuredangle Y(F)_{F=1/8}$ = _____ radians

$$y[n] \xleftrightarrow{\mathcal{F}} \frac{10}{1-0.6e^{-j4\pi F}} = Y(F) \Rightarrow Y(1/8) = \frac{10}{1-0.6e^{-j\pi/2}} = \frac{10}{1+j0.6} = 8.575\angle -0.5404 \text{ radians}$$

2. Given $x[n] \xleftrightarrow[3]{\mathcal{DFT}} X[k]$ and $X[0] = 2$, $X[1] = 3 - j2$, $X[2] = 3 + j2$ find the numerical value of $x[1]$.

$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]e^{j2\pi kn/N} = \frac{1}{3}\sum_{k=0}^{2} X[k]e^{j2\pi kn/3}$$

$$x[1] = \frac{1}{3}\sum_{k=0}^{2} X[k]e^{j2\pi k/3} = \frac{1}{3}\left[2 + (3-j2)e^{j2\pi/3} + (3+j2)e^{j4\pi/3}\right] = 0.8214$$

3. Every DFT harmonic function of a real-valued signal has complex-conjugate symmetry, meaning $X[k] = X^*[-k]$. Also every DFT harmonic function is periodic with fundamental period N where N is the number of data values used to find $X[k]$. That is $X[k] = X[k + mN]$ where N is the fundamental period and m is any integer. Fill in the blanks with correct numbers for this DFT harmonic function of a real-valued signal with $N = 8$.

k	0	1	2	3	4	5	6	7
$X[k]$	5	_____	$2-j7$	$4+j2$	–3	_____	_____	$9+j4$

k	11	–9	26	–47
$X[k]$	_____	_____	_____	_____

k	0	1	2	3	4	5	6	7
$X[k]$	5	$9-j4$	$2-j7$	$4+j2$	–3	$4-j2$	$2+j7$	$9+j4$

k	11	–9	26	–47
$X[k]$	$4+j2$	$9+j4$	$2-j7$	$9-j4$

Solution of EECS 315 Test 12 F13

1. Given the DTFT pair $x[n] \overset{\mathcal{F}}{\longleftrightarrow} \frac{7}{1-0.3e^{-j2\pi F}}$ and $y[n] = \begin{cases} x[n/2] & , n/2 \text{ an integer} \\ 0 & , \text{ otherwise} \end{cases}$, find the numerical magnitude and phase (in radians) of $Y(F)_{F=1/8}$.

$\left|Y(F)_{F=1/8}\right|$ = _____ , $\measuredangle Y(F)_{F=1/8}$ = _____ radians

$$y[n] \overset{\mathcal{F}}{\longleftrightarrow} \frac{7}{1-0.3e^{-j4\pi F}} = Y(F) \Rightarrow Y(1/8) = \frac{7}{1-0.3e^{-j\pi/2}} = \frac{7}{1+j0.3} = 6.7048\angle -0.2915 \text{ radians}$$

2. Given $x[n] \overset{\mathcal{DFT}}{\underset{3}{\longleftrightarrow}} X[k]$ and $X[0] = 2$, $X[1] = 4-j$, $X[2] = 4+j$ find the numerical value of $x[1]$.

$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]e^{j2\pi kn/N} = \frac{1}{3}\sum_{k=0}^{2} X[k]e^{j2\pi kn/3}$$

$$x[1] = \frac{1}{3}\sum_{k=0}^{2} X[k]e^{j2\pi k/3} = \frac{1}{3}\left[2 + (4-j)e^{j2\pi/3} + (4+j)e^{j4\pi/3}\right] = -0.08932$$

3. Every DFT harmonic function of a real-valued signal has complex-conjugate symmetry, meaning $X[k] = X^*[-k]$. Also every DFT harmonic function is periodic with fundamental period N where N is the number of data values used to find $X[k]$. That is $X[k] = X[k+mN]$ where N is the fundamental period and m is any integer. Fill in the blanks with correct numbers for this DFT harmonic function of a real-valued signal with $N = 8$.

k	0	1	2	3	4	5	6	7
$X[k]$	5	_____	$5-j3$	$8+j6$	-2	_____	_____	$11-j2$

k	11	-9	26	-47
$X[k]$	_____	_____	_____	_____

k	0	1	2	3	4	5	6	7
$X[k]$	5	$11+j2$	$5-j3$	$8+j6$	-2	$8-j6$	$5+j3$	$11-j2$

k	11	-9	26	-47
$X[k]$	$8+j6$	$11-j2$	$5-j3$	$11+j2$

Solution of EECS 315 Test 12 F13

1. Given the DTFT pair $x[n] \overset{\mathcal{F}}{\longleftrightarrow} \frac{17}{1-0.5e^{-j2\pi F}}$ and $y[n] = \begin{cases} x[n/2] & , n/2 \text{ an integer} \\ 0 & , \text{otherwise} \end{cases}$, find the numerical magnitude and phase (in radians) of $Y(F)_{F=1/8}$.

$\left|Y(F)_{F=1/8}\right| =$ _____ , $\measuredangle Y(F)_{F=1/8} =$ _____ radians

$$y[n] \overset{\mathcal{F}}{\longleftrightarrow} \frac{17}{1-0.5e^{-j4\pi F}} = Y(F) \Rightarrow Y(1/8) = \frac{17}{1-0.5e^{-j\pi/2}} = \frac{17}{1+j0.5} = 15.2053\angle -0.4637 \text{ radians}$$

2. Given $x[n] \overset{\mathcal{DFT}}{\underset{3}{\longleftrightarrow}} X[k]$ and $X[0] = -1$, $X[1] = 5 - j8$, $X[2] = 5 + j8$ find the numerical value of $x[1]$.

$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]e^{j2\pi kn/N} = \frac{1}{3}\sum_{k=0}^{2} X[k]e^{j2\pi kn/3}$$

$$x[1] = \frac{1}{3}\sum_{k=0}^{2} X[k]e^{j2\pi k/3} = \frac{1}{3}\left[-1+(5-j8)e^{j2\pi/3}+(5+j8)e^{j4\pi/3}\right] = 2.619$$

3. Every DFT harmonic function of a real-valued signal has complex-conjugate symmetry, meaning $X[k] = X^*[-k]$. Also every DFT harmonic function is periodic with fundamental period *N* where *N* is the number of data values used to find $X[k]$. That is $X[k] = X[k+mN]$ where *N* is the fundamental period and *m* is any integer. Fill in the blanks with correct numbers for this DFT harmonic function of a real-valued signal with $N = 8$.

k	0	1	2	3	4	5	6	7
$X[k]$	9	_____	$11-j6$	$3+j7$	-1	_____	_____	$12-j8$

k	11	–9	26	–47
$X[k]$	_____	_____	_____	_____

k	0	1	2	3	4	5	6	7
$X[k]$	9	$12+j8$	$11-j6$	$3+j7$	-1	$3-j7$	$11+j6$	$12-j8$

k	11	–9	26	–47
$X[k]$	$3+j7$	$12-j8$	$11-j6$	$12+j8$