Solution of ECE 315 Test 10 F04

1. The DTFT of $x[n] = 2\delta[n+3] - 3\delta[n-3]$ can be expressed in the form, $X(F) = A\sin(bF) + Ce^{dF}$. Find the numerical values of *A, b, C* and *d.*

$A = \underline{j4}$, $b = \underline{6\pi}$, $C = \underline{-1}$, $d = \underline{-j6\pi}$

$$X(F) = 2e^{+j6\pi F} - 3e^{-j6\pi F} = 2e^{+j6\pi F} - 2e^{-j6\pi F} - e^{-j6\pi F} = j4\sin(6\pi F) - e^{-j6\pi F}$$

2. Let $x[n]$ be a DT signal and let $y[n] = \sum_{m=-\infty}^{n} x[m]$. If $Y(j\Omega) = \cos(2\Omega)$, $x[n]$ consists of exactly four DT impulses. What are their numerical strengths and locations?

Impulse #1. Strength = 1/2 Located at *n* = -2

Impulse #2. Strength = 1/2 Located at *n* = 2

Impulse #3. Strength = -1/2 Located at *n* = 3

Impulse #4. Strength = -1/2 Located at *n* = -1

$$Y(j\Omega) = \cos(2\Omega) = \frac{X(j\Omega)}{1 - e^{-j\Omega}} \Rightarrow X(j\Omega) = \cos(2\Omega)\left(1 - e^{-j\Omega}\right)$$

$$X(j\Omega) = \frac{1}{2}\left(e^{j2\Omega} + e^{-j2\Omega}\right)\left(1 - e^{-j\Omega}\right) = \frac{1}{2}\left(e^{j2\Omega} + e^{-j2\Omega} - e^{j\Omega} - e^{-j3\Omega}\right)$$

$$x[n] = \frac{1}{2}\left(\delta[n+2] + \delta[n-2] - \delta[n+1] - \delta[n-3]\right)$$

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1. The DTFT of $x[n] = 3\delta[n+5] - 5\delta[n-5]$ can be expressed in the form, $X(F) = A\sin(bF) + Ce^{dF}$. Find the numerical values of *A, b, C* and *d.*

$A = \underline{j6}$, $b = \underline{10\pi}$, $C = \underline{-2}$, $d = \underline{-j10\pi}$

$$X(F) = 3e^{+j10\pi F} - 5e^{-j10\pi F} = 3e^{+j10\pi F} - 3e^{-j10\pi F} - 2e^{-j10\pi F} = j6\sin(10\pi F) - 2e^{-j10\pi F}$$

2. Let $x[n]$ be a DT signal and let $y[n] = \sum_{m=-\infty}^{n} x[m]$. If $Y(j\Omega) = 2\cos(4\Omega)$, $x[n]$ consists of exactly four DT impulses. What are their numerical strengths and locations?

Impulse #1. Strength = $\underline{1}$ Located at n = $\underline{-4}$

Impulse #2. Strength = $\underline{1}$ Located at n = $\underline{4}$

Impulse #3. Strength = $\underline{-1}$ Located at n = $\underline{5}$

Impulse #4. Strength = $\underline{-1}$ Located at n = $\underline{-3}$

$$Y(j\Omega) = 2\cos(4\Omega) = \frac{X(j\Omega)}{1 - e^{-j\Omega}} \Rightarrow X(j\Omega) = 2\cos(4\Omega)\left(1 - e^{-j\Omega}\right)$$

$$X(j\Omega) = \left(e^{j4\Omega} + e^{-j4\Omega}\right)\left(1 - e^{-j\Omega}\right) = \left(e^{j4\Omega} + e^{-j4\Omega} - e^{j3\Omega} - e^{-j5\Omega}\right)$$

$$x[n] = \left(\delta[n+4] + \delta[n-4] - \delta[n+3] - \delta[n-5]\right)$$