

Solution of ECE 315 Test 9 F05

Find the numerical values of the constants.

$$(a) \quad 15 \operatorname{tri}(3t - 2) \xleftrightarrow{\mathcal{F}} A \operatorname{sinc}^2(af) e^{bf}$$

$$15 \operatorname{tri}(3t - 2) = 15 \operatorname{tri}(3(t - 2/3)) \xleftrightarrow{\mathcal{F}} 5 \operatorname{sinc}^2(f/3) e^{-j2\pi f(2/3)} = A \operatorname{sinc}^2(af) e^{bf}$$

$$5 \operatorname{sinc}^2(f/3) e^{-j2\pi f(2/3)} = A \operatorname{sinc}^2(af) e^{bf} \Rightarrow A = 5, a = 1/3, b = -j4\pi/3 = -j4.189$$

$$(b) \quad A \operatorname{rect}(a(t - b)) \xleftrightarrow{\mathcal{F}} 13 \operatorname{sinc}(f/6) e^{j10f}$$

$$A \operatorname{rect}(a(t - b)) = 78 \operatorname{rect}(6(t + 5/\pi)) \xleftrightarrow{\mathcal{F}} 13 \operatorname{sinc}(f/6) e^{j10f}$$

$$A \operatorname{rect}(a(t - b)) = 78 \operatorname{rect}(6(t + 5/\pi)) \Rightarrow A = 78, a = 6, b = -5/\pi = -1.592$$

$$(c) \quad 5 \operatorname{sinc}(t/4) \xleftrightarrow{\mathcal{F}} A \operatorname{rect}(a\omega) e^{b\omega}$$

$$5 \operatorname{sinc}(t/4) \xleftrightarrow{\mathcal{F}} 20 \operatorname{rect}(4\omega/2\pi) = A \operatorname{rect}(a\omega) e^{b\omega}$$

$$20 \operatorname{rect}(2\omega/\pi) = A \operatorname{rect}(a\omega) e^{b\omega} \Rightarrow A = 20, a = 2/\pi = 0.6366, b = 0$$

$$(d) \quad A \operatorname{sinc}(a(t - b)) \xleftrightarrow{\mathcal{F}} 11 \operatorname{rect}(2\omega) e^{-j4\omega}$$

$$A \operatorname{sinc}(a(t - b)) = (11/4\pi) \operatorname{sinc}((t - 4)/4\pi) \xleftrightarrow{\mathcal{F}} 11 \operatorname{rect}(4\pi f) e^{-j8\pi f} = 11 \operatorname{rect}(2\omega) e^{-j4\omega}$$

$$A \operatorname{sinc}(a(t - b)) = (11/4\pi) \operatorname{sinc}((t - 4)/4\pi) \Rightarrow$$

$$A = 11/4\pi = 0.8754, a = 1/4\pi = 0.0796, b = 4$$

$$(e) \quad 9e^{-7t^2} \xleftrightarrow{\mathcal{F}} Ae^{-af^2}$$

$$e^{-\pi t^2} \xleftrightarrow{\mathcal{F}} e^{-\pi f^2}$$

Make the transformation $t \rightarrow \sqrt{7/\pi} t$. Then

$$e^{-\pi(\sqrt{7/\pi} t)^2} \xleftrightarrow{\mathcal{F}} \sqrt{\pi/7} e^{-\pi(\sqrt{\pi/7} f)^2}$$

$$e^{-7t^2} \xleftrightarrow{\mathcal{F}} \sqrt{\pi/7} e^{-(\pi f)^2/7}$$

$$9e^{-7t^2} \xleftrightarrow{\mathcal{F}} 9\sqrt{\pi/7} e^{-(\pi f)^2/7} = Ae^{-af^2} \Rightarrow A = 9\sqrt{\pi/7} = 6.029, a = \pi^2/7 = 1.41$$

Solution of ECE 315 Test 9 F05

Find the numerical values of the constants.

$$\begin{aligned} \text{(a)} \quad & A \operatorname{rect}(a(t-b)) \xleftrightarrow{\mathcal{F}} -9 \operatorname{sinc}(f/2) e^{j7f} \\ & A \operatorname{rect}(a(t-b)) = -18 \operatorname{rect}(2(t+7/2\pi)) \xleftrightarrow{\mathcal{F}} -9 \operatorname{sinc}(f/2) e^{j7f} \\ A \operatorname{rect}(a(t-b)) = -18 \operatorname{rect}(2(t+7/2\pi)) & \Rightarrow A = -18, a = 2, b = -7/2\pi = -1.141 \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad & 2 \operatorname{sinc}(t/7) \xleftrightarrow{\mathcal{F}} A \operatorname{rect}(a\omega) e^{b\omega} \\ & 2 \operatorname{sinc}(t/7) \xleftrightarrow{\mathcal{F}} 14 \operatorname{rect}(7\omega/2\pi) = A \operatorname{rect}(a\omega) e^{b\omega} \\ 14 \operatorname{rect}(7\omega/2\pi) = A \operatorname{rect}(a\omega) e^{b\omega} & \Rightarrow A = 14, a = 7/2\pi = 1.141, b = 0 \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad & A \operatorname{sinc}(a(t-b)) \xleftrightarrow{\mathcal{F}} 3 \operatorname{rect}(8\omega) e^{-j2\omega} \\ A \operatorname{sinc}(a(t-b)) = (3/16\pi) \operatorname{sinc}((t-2)/16\pi) & \xleftrightarrow{\mathcal{F}} 3 \operatorname{rect}(16\pi f) e^{-j4\pi f} = 3 \operatorname{rect}(8\omega) e^{-j2\omega} \\ A \operatorname{sinc}(a(t-b)) = (3/16\pi) \operatorname{sinc}((t-2)/16\pi) & \Rightarrow \\ A = 3/16\pi = 0.0597, a = 1/16\pi = 0.0199, b = 2 & \end{aligned}$$

$$\begin{aligned} \text{(d)} \quad & 4e^{-3t^2} \xleftrightarrow{\mathcal{F}} Ae^{-af^2} \\ & e^{-\pi t^2} \xleftrightarrow{\mathcal{F}} e^{-\pi f^2} \\ \text{Make the transformation } t & \rightarrow \sqrt{3/\pi} t. \text{ Then} \\ e^{-\pi(\sqrt{3/\pi} t)^2} & \xleftrightarrow{\mathcal{F}} \sqrt{\pi/3} e^{-\pi(\sqrt{\pi/3} f)^2} \\ e^{-3t^2} & \xleftrightarrow{\mathcal{F}} \sqrt{\pi/3} e^{-(\pi f)^2/3} \\ 4e^{-3t^2} \xleftrightarrow{\mathcal{F}} 4\sqrt{\pi/3} e^{-(\pi f)^2/3} & = Ae^{-af^2} \Rightarrow A = 4\sqrt{\pi/3} = 4.093, a = \pi^2/3 = 3.29 \end{aligned}$$

$$\begin{aligned} \text{(e)} \quad & 6 \operatorname{tri}(2t-3) \xleftrightarrow{\mathcal{F}} A \operatorname{sinc}^2(af) e^{bf} \\ 6 \operatorname{tri}(2t-3) = 6 \operatorname{tri}(2(t-3/2)) & \xleftrightarrow{\mathcal{F}} 3 \operatorname{sinc}^2(f/2) e^{-j2\pi f(3/2)} = A \operatorname{sinc}^2(af) e^{bf} \\ 3 \operatorname{sinc}^2(f/2) e^{-j3\pi f} = A \operatorname{sinc}^2(af) e^{bf} & \Rightarrow A = 3, a = 1/2, b = -j3\pi = -j9.425 \end{aligned}$$