# Solution of EECS 315 Test 7 F13

1. A continuous-time system is described by $4y'(t)+5y(t)=x(t)$ where x is the excitation and y is the response. The impulse response can be written as $h(t)=Ke^{st}u(t)$. Find the numerical values of $K$ and $s$.

   The eigenvalue is the solution of $4s+5=0$ which is $s=-5/4$ .

   Integrating the system equation between $0^-$ and $0^+$ for the special case of $x(t)=\delta(t)$ we get

   $$4K=1 \Rightarrow K=1/4 \ .$$

2. A discrete-time system is described by $7y[n]-5[n-1]=x[n]$ where x is the excitation and y is the response. The impulse response can be written as $h[n]=Kz^n u[n]$. Find the numerical values of $K$ and $z$.

   The eigenvalue is the solution of $7z-5=0$ which is $z=5/7$ .

   $$7h[n]-5h[n-1]=\delta[n] \Rightarrow 7h[0]-5h[-1]=1 \Rightarrow h[0]=1/7=Kz^0 u[0]=K$$

   Therefore $K=1/7$ .

3. If $x(t)=\text{rect}\left(\frac{t-1}{4}\right)*\left[2\delta(t-3)+3\delta(t+1)\right]$ , find the numerical value of $x(1)$ .

$$x(t)=2\,\text{rect}\left(\frac{t-1}{4}\right)*\delta(t-3)+3\,\text{rect}\left(\frac{t-1}{4}\right)*\delta(t+1)$$

$$x(t)=2\,\text{rect}\left(\frac{t-4}{4}\right)+3\,\text{rect}\left(\frac{t}{4}\right)\Rightarrow x1(t)=2\underbrace{\text{rect}\left(\frac{-3}{4}\right)}_{=0}+3\underbrace{\text{rect}\left(\frac{1}{4}\right)}_{=1}=3$$

4. A discrete-time system is described by $20\,y[n]-4\,y[n-1]-3\,y[n-2]=x[n]$ where x is the excitation and y is the response. The impulse response can be written as $h[n]=\left(K_1 z_1^n+K_2 z_2^n\right)u[n]$. Find the numerical values of $K_1$ , $K_2$ , $z_1$ and $z_2$ .

The eigenvalues are the solutions of $20z^2-4z-3=0$ which are $z=0.5$ and $z=-0.3$ .

$$20\,h[n]-4\,h[n-1]-3\,y[n-2]=\delta[n]\Rightarrow 20\,h[0]-4\,h[-1]-3h[-2]=1\Rightarrow h[0]=1/20=\left(K_1 z_1^0+K_2 z_2^0\right)u[0]=K_1+K_2$$

$$20\,h[1]-4\underbrace{h[0]}_{=1/20}-3h[-1]=0\Rightarrow h[1]=1/100=\left(K_1 z_1^1+K_2 z_2^1\right)u[0]=0.5K_1-0.3K_2$$

Solving simultaneously, $K_1=0.03125$ and $K_2=0.01875$

# Solution of EECS 315 Test 7 F13

1. A continuous-time system is described by $3y'(t)+8y(t)=x(t)$ where x is the excitation and y is the response. The impulse response can be written as $h(t)=Ke^{st}u(t)$. Find the numerical values of $K$ and $s$.

   The eigenvalue is the solution of $3s+8=0$ which is $s=-8/3$ .

   Integrating the system equation between $0^-$ and $0^+$ for the special case of $x(t)=\delta(t)$ we get

$$3K=1 \Rightarrow K=1/3 \ .$$

2. A discrete-time system is described by $10y[n]-3[n-1]=x[n]$ where x is the excitation and y is the response. The impulse response can be written as $h[n]=Kz^n u[n]$. Find the numerical values of $K$ and $z$.

   The eigenvalue is the solution of $10z-3=0$ which is $z=3/10$ .

$$10h[n]-3h[n-1]=\delta[n] \Rightarrow 10h[0]-3h[-1]=1 \Rightarrow h[0]=1/10=Kz^0 u[0]=K$$

   Therefore $K=1/10$ .

3. If $\mathrm{x}(t)=\mathrm{rect}\left(\frac{t+1}{4}\right)*\left[2\delta(t-2)+3\delta(t+1)\right]$ , find the numerical value of $\mathrm{x}(1)$ .

$$\mathrm{x}(t)=2\,\mathrm{rect}\left(\frac{t+1}{4}\right)*\delta(t-2)+3\,\mathrm{rect}\left(\frac{t+1}{4}\right)*\delta(t+1)$$

$$\mathrm{x}(t)=2\,\mathrm{rect}\left(\frac{t-1}{4}\right)+3\,\mathrm{rect}\left(\frac{t+2}{4}\right)\Rightarrow \mathrm{x}(1)=2\underbrace{\mathrm{rect}\left(\frac{0}{4}\right)}_{=1}+3\underbrace{\mathrm{rect}\left(\frac{3}{4}\right)}_{=0}=2$$

4. A discrete-time system is described by $10\,\mathrm{y}[n]-\mathrm{y}[n-1]-2\,\mathrm{y}[n-2]=\mathrm{x}[n]$ where x is the excitation and y is the response. The impulse response can be written as $\mathrm{h}[n]=\left(K_1z_1^n+K_2z_2^n\right)\mathrm{u}[n]$. Find the numerical values of $K_1$ , $K_2$ , $z_1$ and $z_2$ .

The eigenvalues are the solutions of $10z^2-z-2=0$ which are $z=0.5$ and $z=-0.4$ .

$$10\,\mathrm{h}[n]-\mathrm{h}[n-1]-2\,\mathrm{y}[n-2]=\delta[n]\Rightarrow 10\,\mathrm{h}[0]-\mathrm{h}[-1]-2\,\mathrm{h}[-2]=1\Rightarrow \mathrm{h}[0]=1/10=\left(K_1z_1^0+K_2z_2^0\right)\mathrm{u}[0]=K_1+K_2$$

$$10\,\mathrm{h}[1]-\underbrace{\mathrm{h}[0]}_{=1/10}-2\,\mathrm{h}[-1]=0\Rightarrow \mathrm{h}[1]=1/100=\left(K_1z_1^1+K_2z_2^1\right)\mathrm{u}[0]=0.5K_1-0.4K_2$$

Solving simultaneously, $K_1=0.05556$ and $K_2=0.04444$

# Solution of EECS 315 Test 7 F13

1. A continuous-time system is described by $9\,y'(t)+2\,y(t)=x(t)$ where x is the excitation and y is the response. The impulse response can be written as $h(t)=Ke^{st}\,u(t)$. Find the numerical values of $K$ and $s$.

   The eigenvalue is the solution of $9s+2=0$ which is $s=-2/9$ .

   Integrating the system equation between $0^-$ and $0^+$ for the special case of $x(t)=\delta(t)$ we get

$$9K=1 \Rightarrow K=1/9 \ .$$

2. A discrete-time system is described by $11y[n]-4[n-1]=x[n]\,11y[n]-4[n-1]=x[n]$ where x is the excitation and y is the response. The impulse response can be written as $h[n]=Kz^n\,u[n]$. Find the numerical values of $K$ and $z$.

   The eigenvalue is the solution of $11z-4=0$ which is $z=4/11$ .

$$11h[n]-4h[n-1]=\delta[n] \Rightarrow 11h[0]-4h[-1]=1 \Rightarrow h[0]=1/11=Kz^0\,u[0]=K$$

   Therefore $K=1/11$ .

3. If $\mathrm{x}(t) = \mathrm{rect}\left(\frac{t-3}{8}\right) * \left[2\delta(t-5) + 7\delta(t+2)\right]$ , find the numerical value of $\mathrm{x}(1)$ .

$$\mathrm{x}(t) = 2\,\mathrm{rect}\left(\frac{t-3}{8}\right) * \delta(t-5) + 7\,\mathrm{rect}\left(\frac{t-3}{8}\right) * \delta(t+2)$$

$$\mathrm{x}(t) = 2\,\mathrm{rect}\left(\frac{t-8}{8}\right) + 7\,\mathrm{rect}\left(\frac{t-1}{4}\right) \Rightarrow \mathrm{x}(1) = 2\underbrace{\mathrm{rect}\left(\frac{-7}{8}\right)}_{=0} + 7\underbrace{\mathrm{rect}\left(\frac{0}{8}\right)}_{=1} = 7$$

4. A discrete-time system is described by $50\,\mathrm{y}[n] + 25\,\mathrm{y}[n-1] + 2\,\mathrm{y}[n-2] = \mathrm{x}[n]$ where x is the excitation and y is the response. The impulse response can be written as $\mathrm{h}[n] = \left(K_1 z_1^n + K_2 z_2^n\right)\mathrm{u}[n]$. Find the numerical values of $K_1$ , $K_2$ , $z_1$ and $z_2$ .

The eigenvalues are the solutions of $50z^2 + 25z + 2 = 0$ which are $z = -0.4$ and $z = -0.1$ .

$$50\,\mathrm{h}[n] + 25\,\mathrm{h}[n-1] + 2\,\mathrm{y}[n-2] = \delta[n] \Rightarrow 50\,\mathrm{h}[0] + 25\,\mathrm{h}[-1] + 2\,\mathrm{h}[-2] = 1 \Rightarrow \mathrm{h}[0] = 1/50 = \left(K_1 z_1^0 + K_2 z_2^0\right)\mathrm{u}[0] = K_1 + K_2$$

$$50\,\mathrm{h}[1] + 25\underbrace{\mathrm{h}[0]}_{=1/50} + 2\,\mathrm{h}[-1] = 0 \Rightarrow \mathrm{h}[1] = -1/100 = \left(K_1 z_1^1 + K_2 z_2^1\right)\mathrm{u}[0] = -0.4K_1 - 0.1K_2$$

Solving simultaneously, $K_1 = 0.02667$ and $K_2 = -0.00667$