Solution to ECE 315 Test #4 F04

In the DT system diagram below, $a = \sqrt{2}$ and $b = 1$.

(a) What is the numerical value of the impulse response at time, $n = 0$? $\mathrm{h}[0] = 1$

The difference equation is $\mathrm{y}[n] + \sqrt{2}\mathrm{y}[n-1] + \mathrm{y}[n-2] = \mathrm{x}[n]$.

For impulse response the equation becomes $\mathrm{h}[n] + \sqrt{2}\mathrm{h}[n-1] + \mathrm{h}[n-2] = \delta[n]$

At time, $n = 0$, the equation is $\mathrm{h}[0] + \sqrt{2}\mathrm{h}[0-1] + \mathrm{h}[0-2] = \mathrm{h}[0] + \sqrt{2}\,(0) + (0) = 1$

(b) What is the numerical value of the impulse response at time, $n = 1$? $\mathrm{h}[1] = -\sqrt{2} = -1.414$

At time, $n = 0$, the equation is $\mathrm{h}[1] + \sqrt{2}\mathrm{h}[1-1] + \mathrm{h}[1-2] = \mathrm{h}[1] + \sqrt{2}\,(1) + (0) = 0$

(c) What are the numerical magnitudes and angles (in radians) of the eigenvalues?

The characteristic equation is $\alpha^2 + \sqrt{2}\alpha + 1 = 0$. The eigenvalues are the roots which are $-0.7071 \pm j0.7071$.

$|\alpha_1| = 1$, $\angle\alpha_1 = \frac{3\pi}{4} = 2.3562$, $|\alpha_2| = 1$, $\angle\alpha_2 = -\frac{3\pi}{4} = -2.3562$

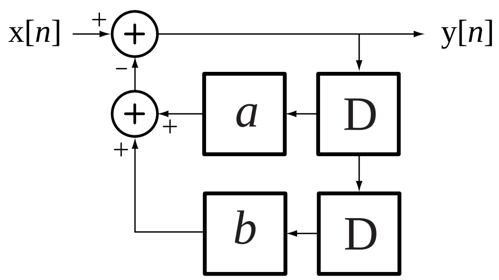
(d) What is the numerical value of the impulse response at time, $n = 223$? $\mathrm{h}[223] = 0$

We need the impulse response in closed form for this answer. The form is $\mathrm{h}[n] = K_1\alpha_1^n + K_2\alpha_2^n$. The two equations for the two constants are $K_1 + K_2 = 1$ and $K_1\alpha_1 + K_2\alpha_2 = -\sqrt{2}$. The solution of these simultaneous equations is $K_1 = 0.5 + j0.5$ and $K_2 = 0.5 - j0.5$. So $\mathrm{h}[n] = (0.5 + j0.5)\, e^{j3\pi n/4} + (0.5 - j0.5)\, e^{-j3\pi n/4}$.

Therefore $\mathrm{h}[223] = (0.5 + j0.5)\, e^{j3\pi(223)/4} + (0.5 - j0.5)\, e^{-j3\pi(223)/4}$.

$\mathrm{h}[223] = 0.707\left(e^{+j\pi/4}e^{j669\pi/4} + e^{-j\pi/4}e^{-j669\pi/4}\right) = 0.707\left(e^{j167.5\pi} + e^{-j167.5\pi}\right) = 0$

(e) Is the system BIBO stable or BIBO unstable? BIBO Unstable



Solution to ECE 315 Test #4 F04

In the DT system diagram below, $a = -\sqrt{2}$ and $b = 1$.

(a) What is the numerical value of the impulse response at time, $n = 0$? $\mathrm{h}[0] = 1$

The difference equation is $\mathrm{y}[n] - \sqrt{2}\mathrm{y}[n-1] + \mathrm{y}[n-2] = \mathrm{x}[n]$.

For impulse response the equation becomes $\mathrm{h}[n] - \sqrt{2}\mathrm{h}[n-1] + \mathrm{h}[n-2] = \delta[n]$

At time, $n = 0$, the equation is $\mathrm{h}[0] - \sqrt{2}\mathrm{h}[0-1] + \mathrm{h}[0-2] = \mathrm{h}[0] - \sqrt{2}\,(0) + (0) = 1$

(b) What is the numerical value of the impulse response at time, $n = 1$? $\mathrm{h}[1] = \sqrt{2} = 1.414$

At time, $n = 0$, the equation is $\mathrm{h}[1] - \sqrt{2}\mathrm{h}[1-1] + \mathrm{h}[1-2] = \mathrm{h}[1] - \sqrt{2}\,(1) + (0) = 0$

(c) What are the numerical magnitudes and angles (in radians) of the eigenvalues?

The characteristic equation is $\alpha^2 - \sqrt{2}\alpha + 1 = 0$. The eigenvalues are the roots which are $0.7071 \pm j0.7071$.

$|\alpha_1| = 1$, $\angle\alpha_1 = \frac{\pi}{4} = 0.7854$, $|\alpha_2| = 1$, $\angle\alpha_2 = -\frac{\pi}{4} = -0.7854$

(d) What is the numerical value of the impulse response at time, $n = 224$? $\mathrm{h}[224] = 1$

We need the impulse response in closed form for this answer. The form is $\mathrm{h}[n] = K_1\alpha_1^n + K_2\alpha_2^n$. The two equations for the two constants are $K_1 + K_2 = 1$ and $K_1\alpha_1 + K_2\alpha_2 = \sqrt{2}$. The solution of these simultaneous equations is $K_1 = 0.5 - j0.5$ and $K_2 = 0.5 + j0.5$. So $\mathrm{h}[n] = (0.5 - j0.5)\,e^{j\pi n/4} + (0.5 + j0.5)\,e^{-j\pi n/4}$.

Therefore $\mathrm{h}[224] = (0.5 - j0.5)\,e^{j3\pi(224)/4} + (0.5 + j0.5)\,e^{-j3\pi(224)/4}$.

$\mathrm{h}[223] = 0.707\left(e^{j\pi/4}e^{j672\pi/4} + e^{+j\pi/4}e^{-j672\pi/4}\right) = 0.707\left(e^{j168.25\pi} + e^{-j168.25\pi}\right) = 1$

(e) Is the system BIBO stable or BIBO unstable? BIBO Unstable