

Solution to ECE 315 Test #4 F04

1. The impulse response of a CT system is $h(t) = 10e^{-4t}u(t) + 3\delta(t)$.

(a) What is the numerical value of the integral of $h(t)$ from 0^- to 0^+ ? Since the range of integration is infinitesimal, the area under $10e^{-4t}u(t)$ is zero because its value is finite at zero. The area under the impulse is, by the definition of the impulse, 3.

$$\int_{0^-}^{0^+} h(t)dt = 3$$

(b) Let $g(t) = \int_{-\infty}^t h(\lambda)d\lambda$. What is the numerical value of the integral of $g(t)$ from 0^- to 0^+ ? The integral of $10e^{-4t}u(t)$ is finite and continuous at $t = 0$. Therefore the area under the integral of $10e^{-4t}u(t)$ is zero in that infinitesimal interval. The integral of the unit impulse is the unit step. Therefore $\int_{-\infty}^t 3\delta(\lambda)d\lambda = 3u(t)$ and $\int_{0^-}^{0^+} 3u(t)dt = 0$. Therefore the integral of $g(t)$ over the infinitesimal interval is zero.

$$\int_{0^-}^{0^+} g(t)dt = 0$$

2. Let $x(t) = \text{tri}(\frac{t-1}{2})$ and let $h(t) = \text{rect}(t+2)$. Let $y(t) = x(t) * h(t)$.

(a) Find the range of times, t , over which $x(t)$ is not zero. The triangle function is centered at $t = 1$ and has a base width of 4. Therefore it extends from -1 to 3. $-1 < t < 3$

(b) Find the range of times, t , over which $h(t)$ is not zero. The rectangle function has is centered at $t = -2$ and has a width of 1. Therefore it extends from -2.5 to -1.5. $-2.5 < t < -1.5$

(c) Find the range of times, t , over which $y(t)$ is not zero. If we time-invert the rectangle function it will then occupy the region of time, 1.5 to 2.5. That is its $t = 0$ position. If we then imagine t increasing from negative infinity, the first time the convolution is not zero occurs when the right edge of the shifted rectangle first strikes the left corner of the triangle. That occurs at $t = -3.5$. Then the area under the product of $x(t)$ and $h(t)$ is non-zero until the left edge of the rectangle gets to the right corner of the triangle. That occurs when $t = 1.5$. Therefore the answer is $-3.5 < t < 1.5$. The base width of $y(t)$ is then 5 which is the sum of the base widths of the triangle and the rectangle as it should be.

(d) What is the maximum numerical value of $y(t)$ over all time, t and at what time, t , does it occur? The maximum area under the product occurs when the time-inverted-and-shifted rectangle, $\text{rect}(t - \tau + 2)$, is centered on the peak of the triangle function which is at $\tau = 1$. The center of the time-inverted-and-unshifted rectangle, $\text{rect}(-\tau + 2)$ is at $\tau = 2$. Therefore to make the center of the rectangle coincide with the peak of the triangle the shift, t , must be -1. At that position the area under the product of the two functions is the area of the triangle function in the range, $0.5 < \tau < 1.5$. That area consists of a rectangle of width, 1 and height 0.75 plus the area of a triangle of base width, 1 and height 0.25. That total area is 0.875.

Maximum of $y(t) = 0.875$ and it occurs at $t = -1$

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1. The impulse response of a CT system is $h(t) = 10e^{-4t}u(t) + 5\delta(t)$.

(a) What is the numerical value of the integral of $h(t)$ from 0^- to 0^+ ? Since the range of integration is infinitesimal, the area under $10e^{-4t}u(t)$ is zero because its value is finite at zero. The area under the impulse is, by the definition of the impulse, 5.

$$\int_{0^-}^{0^+} h(t)dt = 5$$

(b) Let $g(t) = \int_{-\infty}^t h(\lambda)d\lambda$. What is the numerical value of the integral of $g(t)$ from 0^- to 0^+ ? The integral of $10e^{-4t}u(t)$ is finite and continuous at $t = 0$. Therefore the area under the integral of $10e^{-4t}u(t)$ is zero in that infinitesimal interval. The integral of the unit impulse is the unit step. Therefore $\int_{-\infty}^t 5\delta(\lambda)d\lambda = 5u(t)$ and $\int_{0^-}^{0^+} 5u(t)dt = 0$. Therefore the integral of $g(t)$ over the infinitesimal interval is zero.

$$\int_{0^-}^{0^+} g(t)dt = 0$$

2. Let $x(t) = \text{tri}(\frac{t+1}{2})$ and let $h(t) = \text{rect}(t-2)$. Let $y(t) = x(t) * h(t)$.

(a) Find the range of times, t , over which $x(t)$ is not zero. The triangle function is centered at $t = -1$ and has a base width of 4. Therefore it extends from -3 to 1. $-3 < t < 1$

(b) Find the range of times, t , over which $h(t)$ is not zero. The rectangle function has is centered at $t = 2$ and has a width of 1. Therefore it extends from 1.5 to 2.5. $1.5 < t < 2.5$

(c) Find the range of times, t , over which $y(t)$ is not zero. If we time-invert the rectangle function it will then occupy the region of time, -2.5 to -1.5. That is its $t = 0$ position. If we then imagine t increasing from negative infinity, the first time the convolution is not zero occurs when the right edge of the shifted rectangle first strikes the left corner of the triangle. That occurs at $t = -1.5$. Then the area under the product of $x(t)$ and $h(t)$ is non-zero until the left edge of the rectangle gets to the right corner of the triangle. That occurs when $t = 3.5$. Therefore the answer is $-1.5 < t < 3.5$. The base width of $y(t)$ is then 5 which is the sum of the base widths of the triangle and the rectangle as it should be.

(d) What is the maximum numerical value of $y(t)$ over all time, t and at what time, t , does it occur? The maximum area under the product occurs when the time-inverted-and-shifted rectangle, $\text{rect}(t - \tau - 2)$, is centered on the peak of the triangle function which is at $\tau = -1$. The center of the time-inverted-and-unshifted rectangle, $\text{rect}(-\tau - 2)$ is at $\tau = -2$.

Therefore to make the center of the rectangle coincide with the peak of the triangle the shift, t , must be $+1$. At that position the area under the product of the two functions is the area of the triangle function in the range, $-1.5 < \tau < -0.5$. That area consists of a rectangle of width, 1 and height 0.75 plus the area of a triangle of base width, 1 and height 0.25. That total area is 0.875.

Maximum of $y(t) = 0.875$ and it occurs at $t = 1$

Solution to ECE 315 Test #4 F04

1. The impulse response of a CT system is $h(t) = 10e^{-4t}u(t) + 8\delta(t)$.

(a) What is the numerical value of the integral of $h(t)$ from 0^- to 0^+ ? Since the range of integration is infinitesimal, the area under $10e^{-4t}u(t)$ is zero because its value is finite at zero. The area under the impulse is, by the definition of the impulse, 8.

$$\int_{0^-}^{0^+} h(t)dt = 8$$

(b) Let $g(t) = \int_{-\infty}^t h(\lambda)d\lambda$. What is the numerical value of the integral of $g(t)$ from 0^- to 0^+ ? The integral of $10e^{-4t}u(t)$ is finite and continuous at $t = 0$. Therefore the area under the integral of $10e^{-4t}u(t)$ is zero in that infinitesimal interval. The integral of the unit impulse is the unit step. Therefore $\int_{-\infty}^t 8\delta(\lambda)d\lambda = 8u(t)$ and $\int_{0^-}^{0^+} 8u(t)dt = 0$. Therefore the integral of $g(t)$ over the infinitesimal interval is zero.

$$\int_{0^-}^{0^+} g(t)dt = 0$$

2. Let $x(t) = \text{tri}(\frac{t-1}{2})$ and let $h(t) = \text{rect}(t+1)$. Let $y(t) = x(t) * h(t)$.

(a) Find the range of times, t , over which $x(t)$ is not zero. The triangle function is centered at $t = 1$ and has a base width of 4. Therefore it extends from -1 to 3. $-1 < t < 3$

(b) Find the range of times, t , over which $h(t)$ is not zero. The rectangle function has is centered at $t = -1$ and has a width of 1. Therefore it extends from -1.5 to -0.5. $-1.5 < t < -0.5$

(c) Find the range of times, t , over which $y(t)$ is not zero. If we time-invert the rectangle function it will then occupy the region of time, 0.5 to 1.5. That is its $t = 0$ position. If we then imagine t increasing from negative infinity, the first time the convolution is not zero occurs when the right edge of the shifted rectangle first strikes the left corner of the triangle. That occurs at $t = -2.5$. Then the area under the product of $x(t)$ and $h(t)$ is non-zero until the left edge of the rectangle gets to the right corner of the triangle. That occurs when $t = 2.5$. Therefore the answer is $-2.5 < t < 2.5$. The base width of $y(t)$ is then 5 which is the sum of the base widths of the triangle and the rectangle as it should be.

(d) What is the maximum numerical value of $y(t)$ over all time, t and at what time, t , does it occur? The maximum area under the product occurs when the time-inverted-and-shifted rectangle, $\text{rect}(t - \tau + 1)$, is centered on the peak of the triangle function which is at $\tau = 1$. The center of the time-inverted-and-unshifted rectangle, $\text{rect}(-\tau + 1)$ is at $\tau = 1$. Therefore to make the center of the rectangle coincide with the peak of the triangle the shift, t , must be 0. At that position the area under the product of the two functions is the area of the triangle function in the range, $0.5 < \tau < 1.5$. That area consists of a rectangle of width, 1 and height 0.75 plus the area of a triangle of base width, 1 and height 0.25. That total area is 0.875.

Maximum of $y(t) = 0.875$ and it occurs at $t = 0$