# Solution of ECE 315 Test 5 F05

1. Let

$$\mathrm{x}_1[n] = 2\delta[n] - 2\delta[n-1] + \delta[n-2] \text{ and } \mathrm{x}_2[n] = -4\delta[n+1] + 3\delta[n-1]$$

and let $\mathrm{x}[n] = \mathrm{x}_1[n] * \mathrm{x}_2[n]$. In the space provided sketch $\mathrm{x}[n]$ for $-4 \le n \le 4$.

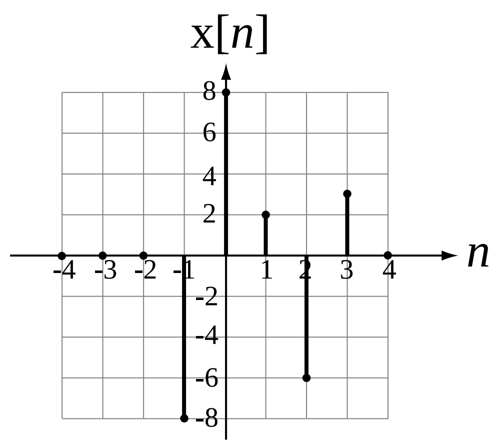
$$\mathrm{x}[n] = (2\delta[n] - 2\delta[n-1] + \delta[n-2]) * (-4\delta[n+1] + 3\delta[n-1])$$

$$\mathrm{x}[n] = \begin{pmatrix} 2\delta[n] * (-4\delta[n+1] + 3\delta[n-1]) \\ -2\delta[n-1] * (-4\delta[n+1] + 3\delta[n-1]) \\ +\delta[n-2] * (-4\delta[n+1] + 3\delta[n-1]) \end{pmatrix}$$

Using $\mathrm{x}[n] * A\delta[n-n_0] = A\,\mathrm{x}[n-n_0]$,

$$\mathrm{x}[n] = -8\delta[n+1] + 6\delta[n-1] + 8\delta[n] - 6\delta[n-2] - 4\delta[n-1] + 3\delta[n-3]$$

$$\mathrm{x}[n] = -8\delta[n+1] + 8\delta[n] + 2\delta[n-1] - 6\delta[n-2] + 3\delta[n-3]$$




Alternate Solution:

$$\mathrm{x}[n] = (2\delta[n] - 2\delta[n-1] + \delta[n-2]) * (-4\delta[n+1] + 3\delta[n-1])$$

$$\mathrm{x}[n] = \sum_{m=-\infty}^{\infty} (2\delta[m] - 2\delta[m-1] + \delta[m-2])(-4\delta[n-m+1] + 3\delta[n-m-1])$$

$$\mathrm{x}[n] = \sum_{m=-\infty}^{\infty} \begin{pmatrix} -8\delta[m]\delta[n-m+1] + 6\delta[m]\delta[n-m-1] + 8\delta[m-1]\delta[n-m+1] \\ -6\delta[m-1]\delta[n-m-1] - 4\delta[m-2]\delta[n-m+1] + 3\delta[m-2]\delta[n-m-1] \end{pmatrix}$$

$$\mathrm{x}[n] = -8\delta[n+1] + 6\delta[n-1] + 8\delta[n] - 6\delta[n-2] - 4\delta[n-1] + 3\delta[n-3]$$

$$\mathrm{x}[n] = -8\delta[n+1] + 8\delta[n] + 2\delta[n-1] - 6\delta[n-2] + 3\delta[n-3]$$

(Could also be done graphically.)

2. A DT system is described by $y[n]+1.8\,y[n-1]+1.2\,y[n-2]=x[n]$.

(a) Is it stable?

The eigenvalues are the solutions of $\alpha^n+1.8\alpha^{n-1}+1.2\alpha^{n-2}=0$. They are $\alpha=-0.9\pm j0.6245$ or $\alpha=1.094e^{\pm j2.535}$. Their magnitudes are both 1.0954 which is greater than one. Therefore the system is unstable.

(b) If $h[n]$ is this system's impulse response, fill in the table below.

| $n$ | 0 | 3 | 5 |
|---|---|---|---|
| $h[n]$ | 1 | –1.512 | 1.322 |

Iterating on the difference equation with $x[n]=\delta[n]$ and $y[n]=h[n]$.

| $n$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $h[n]$ | 1 | –1.8 | 2.04 | –1.512 | 0.2736 | 1.322 |

# Solution of ECE 315 Test 5 F05

1. Let

$$x_1[n]=2\delta[n]+2\delta[n-1]+\delta[n-2] \text{ and } x_2[n]=4\delta[n+1]-3\delta[n-1]$$

and let $x[n]=x_1[n]*x_2[n]$. In the space provided sketch $x[n]$ for $-4\le n\le 4$.

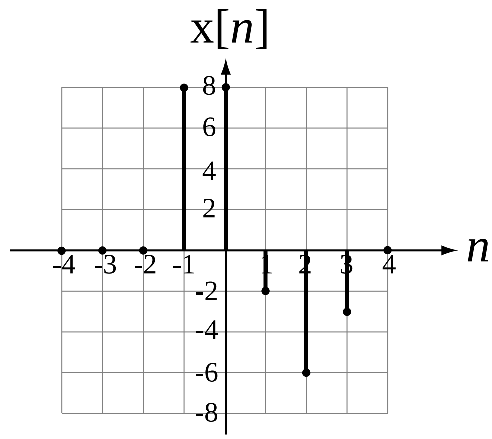
$$x[n]=\left(2\delta[n]+2\delta[n-1]+\delta[n-2]\right)*\left(4\delta[n+1]-3\delta[n-1]\right)$$

$$x[n]=\begin{pmatrix}2\delta[n]*\left(4\delta[n+1]-3\delta[n-1]\right)\\+2\delta[n-1]*\left(4\delta[n+1]-3\delta[n-1]\right)\\+\delta[n-2]*\left(4\delta[n+1]-3\delta[n-1]\right)\end{pmatrix}$$

Using $x[n]*A\delta[n-n_0]=A\,x[n-n_0]$,

$$x[n]=8\delta[n+1]-6\delta[n-1]+8\delta[n]-6\delta[n-2]+4\delta[n-1]-3\delta[n-3]$$

$$x[n]=8\delta[n+1]+8\delta[n]-2\delta[n-1]-6\delta[n-2]-3\delta[n-3]$$




Alternate Solution:

$$\mathrm{x}[n] = (2\delta[n] + 2\delta[n-1] + \delta[n-2]) * (4\delta[n+1] - 3\delta[n-1])$$

$$\mathrm{x}[n] = \sum_{m=-\infty}^{\infty} (2\delta[m] + 2\delta[m-1] + \delta[m-2])(4\delta[n-m+1] - 3\delta[n-m-1])$$

$$\mathrm{x}[n] = \sum_{m=-\infty}^{\infty} \begin{pmatrix} 8\delta[m]\delta[n-m+1] - 6\delta[m]\delta[n-m-1] + 8\delta[m-1]\delta[n-m+1] \\ -6\delta[m-1]\delta[n-m-1] + 4\delta[m-2]\delta[n-m+1] - 3\delta[m-2]\delta[n-m-1] \end{pmatrix}$$

$$\mathrm{x}[n] = 8\delta[n+1] - 6\delta[n-1] + 8\delta[n] - 6\delta[n-2] + 4\delta[n-1] - 3\delta[n-3]$$

$$\mathrm{x}[n] = 8\delta[n+1] + 8\delta[n] - 2\delta[n-1] - 6\delta[n-2] - 3\delta[n-3]$$

(Could also be done graphically.)

2. A DT system is described by $\mathrm{y}[n] - 1.8\,\mathrm{y}[n-1] + 1.2\,\mathrm{y}[n-2] = \mathrm{x}[n]$.

(a) Is it stable?

The eigenvalues are the solutions of $\alpha^n - 1.8\alpha^{n-1} + 1.2\alpha^{n-2} = 0$. They are $\alpha = 0.9 \pm j0.6245$ or $\alpha = 1.094e^{\pm j0.6066}$. Their magnitudes are both 1.0954 which is greater than one. Therefore the system is unstable.

(b) If $\mathrm{h}[n]$ is this system's impulse response, fill in the table below.

| $n$ | 0 | 3 | 5 |
|---|---|---|---|
| $\mathrm{h}[n]$ | 1 | 1.512 | $-1.322$ |

Iterating on the difference equation with $\mathrm{x}[n] = \delta[n]$ and $\mathrm{y}[n] = \mathrm{h}[n]$.

| $n$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $\mathrm{h}[n]$ | 1 | 1.8 | 2.04 | 1.512 | 0.2736 | $-1.322$ |