Solution of ECE 315 Test 2 F06

1. A system is described by the equation $y[n] = e^{x[n]}$ in which x is the excitation and y is the response. Identify its properties by circling the appropriate answer, Yes or No.

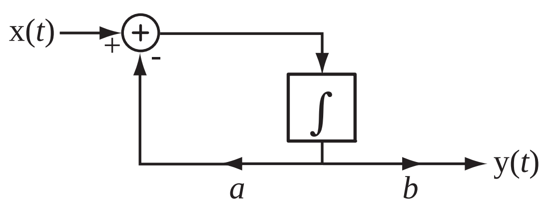
Linear	Yes	**No**	Time Invariant	**Yes**	No
Stable	**Yes**	No	Dynamic	Yes	**No**
Causal	**Yes**	No	Invertible	**Yes**	No

2. In the system below $a = 7$ and $b = 3$.

(a) Write the differential equation describing it.

$$(1/3)y'(t) = x(t) - (7/3)y(t)$$

(b) The impulse response can be written in the form $h(t) = Ke^{\lambda t}u(t)$. Find the numerical values of K and λ.
K = 3 , λ = -7



$$y'(t) + 7y(t) = 3x(t)$$

$$h'(t) + 7h(t) = 3\delta(t)$$

Eigenvalue $\lambda = -7$. Integrating once through zero.

$$\underbrace{h(0^+)}_{=K} - \underbrace{h(0^-)}_{=0} + 7\underbrace{\int_{0^-}^{0^+} h(t)dt}_{=0} = 3\left[\underbrace{u(0^+)}_{=1} - \underbrace{u(0^-)}_{=0}\right] \Rightarrow K = 3$$

3. If $\mathrm{y}(t)=\mathrm{x}(t)*\mathrm{h}(t)$ and $\mathrm{x}(t)=4\,\mathrm{rect}(t-1)$ and $\mathrm{h}(t)=3\,\mathrm{rect}(t)$, what is the numerical value of $\mathrm{y}(1/2)$? $\mathrm{y}(1/2)$ = <u>6</u>

$$\mathrm{y}(t)=\int_{-\infty}^{\infty}\mathrm{x}(\tau)\mathrm{h}(t-\tau)\,d\tau=\int_{-\infty}^{\infty}4\,\mathrm{rect}(\tau-1)3\,\mathrm{rect}(t-\tau)\,d\tau$$

$$\mathrm{y}(1/2)=12\int_{-\infty}^{\infty}\mathrm{rect}(\tau-1)\mathrm{rect}(1/2-\tau)\,d\tau=12\int_{-\infty}^{\infty}\mathrm{rect}(\tau-1)\mathrm{rect}(\tau-1/2)\,d\tau$$

$\mathrm{rect}(\tau-1)$ is 1 between 1/2 and 3/2 and zero elsewhere
$\mathrm{rect}(\tau-1/2)$ is 1 between 0 and 1 and zero elsewhere
$\therefore\ \mathrm{rect}(\tau-1)\mathrm{rect}(\tau-1/2)$ is 1 between 1/2 and 1 and zero elsewhere

$$\mathrm{y}(1/2)=12\int_{1/2}^{1}d\tau=6$$

4. If $\mathrm{y}[n]=\mathrm{x}[n]*\mathrm{h}[n]$ and $\mathrm{x}[n]=\mathrm{ramp}[n]$ and $\mathrm{h}[n]=2\delta[n]-3\delta[n-4]$, what is the numerical value of $\mathrm{y}[10]$?
$\mathrm{y}[10]$ = <u>2</u>

$$\mathrm{y}[n]=\mathrm{ramp}[n]*(2\delta[n]-3\delta[n-4])$$

$$\mathrm{y}[n]=2\,\mathrm{ramp}[n]*\delta[n]-3\,\mathrm{ramp}[n]*\delta[n-4]$$

$$\mathrm{y}[n]=2\,\mathrm{ramp}[n]-3\,\mathrm{ramp}[n-4]$$

$$\mathrm{y}[10]=2\,\mathrm{ramp}[10]-3\,\mathrm{ramp}[6]=20-18=2$$

5. A system with an impulse response $\mathrm{h}_1[n] = a^n \mathrm{u}[n]$ is cascade connected with a second system with impulse response $\mathrm{h}_2[n] = b^n \mathrm{u}[n]$. If $a = 1/2$ and $b = 2/3$, and $\mathrm{h}[n]$ is the impulse response of the overall cascade-connected system what is the numerical value of $\mathrm{h}[3]$?

$\mathrm{h}[3]$ = <u>0.8102</u>

$$\left[\sum_{n=0}^{N-1} r^n = \begin{cases} N & , \ r = 1 \\ \dfrac{1-r^N}{1-r} & , \ r \neq 1 \end{cases}\right]$$

$$\mathrm{h}[n] = \mathrm{h}_1[n] * \mathrm{h}_2[n] = \sum_{m=-\infty}^{\infty} a^m \mathrm{u}[m] b^{n-m} \mathrm{u}[n-m]$$

$$\mathrm{h}[n] = b^n \sum_{m=0}^{n} a^m b^{-m} = b^n \sum_{m=0}^{n} (a/b)^m$$

$$\mathrm{h}[n] = b^n \frac{1-(a/b)^{n+1}}{1-a/b} = \frac{b^n - a^{n+1}/b}{1-a/b} = \frac{b^{n+1} - a^{n+1}}{b-a}$$

$$\mathrm{h}[n] = \frac{(2/3)^{n+1} - (1/2)^{n+1}}{1/6} = 6\left[(2/3)^{n+1} - (1/2)^{n+1}\right]$$

$$\mathrm{h}[3] = 6\left[(2/3)^4 - (1/2)^4\right] = 0.8102$$

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1. A system is described by the equation $y[n] = \ln(x[n-1])$ in which x is the excitation and y is the response. Identify its properties by circling the appropriate answer, Yes or No.

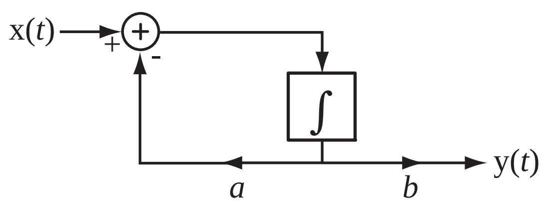
Linear	Yes	[No]	Time Invariant	[Yes]	No
Stable	[Yes]	No	Dynamic	[Yes]	No
Causal	[Yes]	No	Invertible	[Yes]	No

2. In the system below $a = 2$ and $b = 4$.

(a) Write the differential equation describing it.

$$(1/4)y'(t) = x(t) - (1/2)y(t)$$

(b) The impulse response can be written in the form $h(t) = Ke^{\lambda t}u(t)$. Find the numerical values of K and λ.
$K = \underline{4}$, $\lambda = \underline{-2}$



$$y'(t) + 2y(t) = 4x(t)$$

$$h'(t) + 2h(t) = 4\delta(t)$$

Eigenvalue $\lambda = -2$. Integrating once through zero.

$$\underbrace{h(0^+)}_{=K} - \underbrace{h(0^-)}_{=0} + \underbrace{2\int_{0^-}^{0^+} h(t)\,dt}_{=0} = 4\left[\underbrace{u(0^+)}_{=1} - \underbrace{u(0^-)}_{=0}\right] \Rightarrow K = 4$$

3. If $y(t) = x(t) * h(t)$ and $x(t) = 2\,\mathrm{rect}(t+1)$ and $h(t) = 5\,\mathrm{rect}(t)$, what is the numerical value of $y(-1/2)$? $y(-1/2)$ = 5

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau = \int_{-\infty}^{\infty} 2\,\mathrm{rect}(\tau+1)5\,\mathrm{rect}(t-\tau)\,d\tau$$

$$y(-1/2) = 10\int_{-\infty}^{\infty} \mathrm{rect}(\tau+1)\mathrm{rect}(-1/2-\tau)\,d\tau = 10\int_{-\infty}^{\infty} \mathrm{rect}(\tau+1)\mathrm{rect}(\tau+1/2)\,d\tau$$

$\mathrm{rect}(\tau+1)$ is 1 between -1/2 and -3/2 and zero elsewhere

$\mathrm{rect}(\tau+1/2)$ is 1 between 0 and -1 and zero elsewhere

$\therefore\ \mathrm{rect}(\tau+1)\mathrm{rect}(\tau+1/2)$ is 1 between -1/2 and -1 and zero elsewhere

$$y(-1/2) = 10\int_{-1}^{-1/2} d\tau = 5$$

4. If $y[n] = x[n] * h[n]$ and $x[n] = (0.9)^n u[n]$ and $h[n] = 2\delta[n] - 3\delta[n-4]$, what is the numerical value of $y[3]$?

$y[3]$ = 1.458

$$y[n] = (0.9)^n u[n] * (2\delta[n] - 3\delta[n-4])$$

$$y[n] = 2(0.9)^n u[n] * \delta[n] - 3(0.9)^n u[n] * \delta[n-4]$$

$$y[n] = 2(0.9)^n u[n] - 3(0.9)^{n-4} u[n-4]$$

$$y[3] = 2(0.9)^3 u[3] - 3(0.9)^{-1} u[-1] = 2(0.9)^3 = 1.458$$

5. A system with an impulse response $\mathrm{h}_1[n] = a^n \mathrm{u}[n]$ is cascade connected with a second system with impulse response $\mathrm{h}_2[n] = b^n \mathrm{u}[n]$. If $a = 1/4$ and $b = 1/3$, and $\mathrm{h}[n]$ is the impulse response of the overall cascade-connected system what is the numerical value of $\mathrm{h}[3]$?

$\mathrm{h}[3]$ = 0.1013

$$\left[\sum_{n=0}^{N-1} r^n = \begin{cases} N & , \ r = 1 \\ \dfrac{1-r^N}{1-r} & , \ r \neq 1 \end{cases} \right]$$

$$\mathrm{h}[n] = \mathrm{h}_1[n] * \mathrm{h}_2[n] = \sum_{m=-\infty}^{\infty} a^m \mathrm{u}[m] b^{n-m} \mathrm{u}[n-m]$$

$$\mathrm{h}[n] = b^n \sum_{m=0}^{n} a^m b^{-m} = b^n \sum_{m=0}^{n} (a/b)^m$$

$$\mathrm{h}[n] = b^n \frac{1-(a/b)^{n+1}}{1-a/b} = \frac{b^n - a^{n+1}/b}{1-a/b} = \frac{b^{n+1} - a^{n+1}}{b-a}$$

$$\mathrm{h}[n] = \frac{(1/4)^{n+1} - (1/3)^{n+1}}{-1/12} = 12\left[(1/3)^{n+1} - (1/4)^{n+1}\right]$$

$$\mathrm{h}[3] = 12\left[(1/3)^4 - (1/4)^4\right] = 0.1013$$

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1. A system is described by the equation $\mathrm{y}[n] = \ln(\mathrm{x}[n+1])$ in which x is the excitation and y is the response. Identify its properties by circling the appropriate answer, Yes or No.

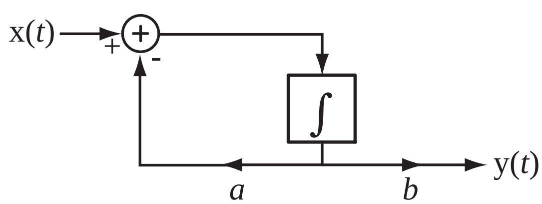
Linear	Yes	**No**	Time Invariant	**Yes**	No
Stable	**Yes**	No	Dynamic	**Yes**	No
Causal	Yes	**No**	Invertible	**Yes**	No

2. In the system below $a = 8$ and $b = 5$.

(a) Write the differential equation describing it.

$$(1/5)\mathrm{y}'(t) = \mathrm{x}(t) - (8/5)\mathrm{y}(t)$$

(b) The impulse response can be written in the form $\mathrm{h}(t) = Ke^{\lambda t}\mathrm{u}(t)$. Find the numerical values of K and λ.
K = <u>5</u> , λ = <u>-8</u>



$$\mathrm{y}'(t) + 8\,\mathrm{y}(t) = 5\,\mathrm{x}(t)$$

$$\mathrm{h}'(t) + 8\,\mathrm{h}(t) = 5\delta(t)$$

Eigenvalue $\lambda = -8$. Integrating once through zero.

$$\underbrace{\mathrm{h}(0^+)}_{=K} - \underbrace{\mathrm{h}(0^-)}_{=0} + \underbrace{8\int_{0^-}^{0^+} \mathrm{h}(t)\,dt}_{=0} = 5\left[\underbrace{\mathrm{u}(0^+)}_{=1} - \underbrace{\mathrm{u}(0^-)}_{=0}\right] \Rightarrow K = 5$$

3. If $y(t) = x(t) * h(t)$ and $x(t) = 2\,\text{rect}(t)$ and $h(t) = -3\,\text{rect}(t/2)$, what is the numerical value of $y(1)$? $y(1)$ = -3

$$y(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau)\, d\tau = \int_{-\infty}^{\infty} 2\,\text{rect}(\tau)(-3)\,\text{rect}((t-\tau)/2)\, d\tau$$

$$y(1) = -6 \int_{-\infty}^{\infty} \text{rect}(\tau)\,\text{rect}((1-\tau)/2)\, d\tau = -6 \int_{-\infty}^{\infty} \text{rect}(\tau)\,\text{rect}((\tau-1)/2)\, d\tau$$

$\text{rect}(\tau)$ is 1 between -1/2 and 1/2 and zero elsewhere
$\text{rect}((\tau-1)/2)$ is 1 between 0 and 2 and zero elsewhere
$\therefore \text{rect}(\tau)\text{rect}(\tau/2)$ is 1 between 0 and 1/2 and zero elsewhere

$$y(1) = -6 \int_{0}^{1/2} d\tau = -3$$

4. If $y[n] = x[n] * h[n]$ and $x[n] = (0.9)^n u[n]$ and $h[n] = 2\delta[n] - 3\delta[n-2]$, what is the numerical value of $y[3]$?
$y[3]$ = -1.242

$$y[n] = (0.9)^n u[n] * (2\delta[n] - 3\delta[n-2])$$

$$y[n] = 2(0.9)^n u[n] * \delta[n] - 3(0.9)^n u[n] * \delta[n-2]$$

$$y[n] = 2(0.9)^n u[n] - 3(0.9)^{n-2} u[n-2]$$

$$y[3] = 2(0.9)^3 u[3] - 3(0.9) u[1] = 2(0.9)^3 - 3(0.9) = -1.242$$

5. A system with an impulse response $h_1[n] = a^n u[n]$ is cascade connected with a second system with impulse response $h_2[n] = b^n u[n]$. If $a = 1/5$ and $b = 1/3$, and $h[n]$ is the impulse response of the overall cascade-connected system what is the numerical value of $h[2]$?

$h[2]$ = 0.2178

$$\left[\sum_{n=0}^{N-1} r^n = \begin{cases} N & , \; r = 1 \\ \dfrac{1-r^N}{1-r} & , \; r \neq 1 \end{cases} \right]$$

$$h[n] = h_1[n] * h_2[n] = \sum_{m=-\infty}^{\infty} a^m u[m] b^{n-m} u[n-m]$$

For $n < 0$, $h[n] = 0$.

For $n \geq 0$,

$$h[n] = b^n \sum_{m=0}^{n} a^m b^{-m} = b^n \sum_{m=0}^{n} (a/b)^m$$

$$h[n] = b^n \frac{1-(a/b)^{n+1}}{1-a/b} = \frac{b^n - a^{n+1}/b}{1-a/b} = \frac{b^{n+1} - a^{n+1}}{b-a}$$

$$h[n] = \frac{(1/5)^{n+1} - (1/3)^{n+1}}{-2/15} = 7.5\left[(1/3)^{n+1} - (1/5)^{n+1}\right]$$

For all time

$$h[n] \backslash = 7.5\left[(1/3)^{n+1} - (1/5)^{n+1}\right] u[n]$$

$$h[2] = 7.5\left[(1/3)^3 - (1/5)^3\right] u[2] = 0.2178$$