Solution of ECE 315 Test 3 F06

1. Find the numerical value of the signal energy of $\mathrm{x}[n] = 2\,\mathrm{rect}_2[n-1] - 3\,\mathrm{tri}(n/3)$.

$$E_x = \sum_{n=-\infty}^{\infty} |\mathrm{x}[n]|^2 = \sum_{n=-\infty}^{\infty} |2\,\mathrm{rect}_2[n-1] - 3\,\mathrm{tri}(n/3)|^2 = \sum_{n=-\infty}^{\infty} [2\,\mathrm{rect}_2[n-1] - 3\,\mathrm{tri}(n/3)]^2$$

First fill in the table below with numbers.

n	–3	–2	–1	0	1	2	3	4
$2\,\mathrm{rect}_2[n-1]$	0	0	2	2	2	2	2	0
$3\,\mathrm{tri}(n/3)$	0	1	2	3	2	1	0	0
$\mathrm{x}[n]$	0	–1	0	–1	0	1	2	0
$\mathrm{x}^2[n]$	0	1	0	1	0	1	4	0

The sum of the squares is 7. Therefore $E_x = 7$.

2. Find the numerical value of the average signal power of a periodic signal $\mathrm{x}(t)$ with period $T_0 = 3$, one period of which is described by

$$\mathrm{x}(t) = \begin{cases} 3\,\mathrm{rect}\left(\dfrac{t-1}{2}\right) - 2\,\mathrm{rect}(t-1/2) & , \ 0 < t < 2 \\ -3 & , \ 2 < t < 3 \end{cases}.$$

$$P_x = \frac{1}{T_0}\int_{T_0} |\mathrm{x}(t)|^2\, dt = \text{average value of } \mathrm{x}^2(t)$$

First fill in the table below with numbers.

t	$0 < t < 1$	$1 < t < 2$	$2 < t < 3$
$3\,\mathrm{rect}\left(\dfrac{t-1}{2}\right)$	3	3	0
$2\,\mathrm{rect}(t-1/2)$	2	0	0
$\mathrm{x}(t)$	1	3	–3
$\mathrm{x}^2(t)$	1	9	9

Total area under $\mathrm{x}^2(t)$ over one period is 19. Therefore the average value of $\mathrm{x}^2(t)$ is 19/3 which is the same as the average signal power P_x.

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1. Find the numerical value of the signal energy of $\mathrm{x}[n] = 5\,\mathrm{rect}_2[n-1] - 6\,\mathrm{tri}(n/3)$.

$$E_x = \sum_{n=-\infty}^{\infty} |\mathrm{x}[n]|^2 = \sum_{n=-\infty}^{\infty} |5\,\mathrm{rect}_2[n-1] - 6\,\mathrm{tri}(n/3)|^2 = \sum_{n=-\infty}^{\infty} [5\,\mathrm{rect}_2[n-1] - 6\,\mathrm{tri}(n/3)]^2$$

First fill in the table below with numbers.

n	–3	–2	–1	0	1	2	3	4
$5\,\mathrm{rect}_2[n-1]$	0	0	5	5	5	5	5	0
$6\,\mathrm{tri}(n/3)$	0	2	4	6	4	2	0	0
$\mathrm{x}[n]$	0	–2	1	–1	1	3	5	0
$\mathrm{x}^2[n]$	0	4	1	1	1	9	25	0

The sum of the squares is 41. Therefore $E_x = 41$.

2. Find the numerical value of the average signal power of a periodic signal $\mathrm{x}(t)$ with period $T_0 = 3$, one period of which is described by

$$\mathrm{x}(t) = \begin{cases} 2\,\mathrm{rect}\left(\frac{t+1}{2}\right) - 3\,\mathrm{rect}(t+3/2) & , \quad -2 < t < 0 \\ -2 & , \quad 0 < t < 1 \end{cases}.$$

$$P_x = \frac{1}{T_0}\int_{T_0} |\mathrm{x}(t)|^2\,dt = \text{average value of } \mathrm{x}^2(t)$$

First fill in the table below with numbers.

t	$-2 < t < -1$	$-1 < t < 0$	$0 < t < 1$
$2\,\mathrm{rect}\left(\frac{t+1}{2}\right)$	2	2	0
$3\,\mathrm{rect}(t+3/2)$	3	0	0
$\mathrm{x}(t)$	–1	2	–2
$\mathrm{x}^2(t)$	1	4	4

Total area under $\mathrm{x}^2(t)$ over one period is 9. Therefore the average value of $\mathrm{x}^2(t)$ is 9/3=3 which is the same as the average signal power P_x.

Solution of ECE 315 Test 3 F06

1. Find the numerical value of the signal energy of $x[n] = -3\,\text{rect}_2[n-2] + 6\,\text{tri}(n/3)$.

$$E_x = \sum_{n=-\infty}^{\infty} \left|x[n]\right|^2 = \sum_{n=-\infty}^{\infty} \left|-3\,\text{rect}_2[n-2] + 6\,\text{tri}(n/3)\right|^2 = \sum_{n=-\infty}^{\infty} \left[-3\,\text{rect}_2[n-2] + 6\,\text{tri}(n/3)\right]^2$$

First fill in the table below with numbers.

n	–3	–2	–1	0	1	2	3	4
$3\,\text{rect}_2[n-2]$	0	0	0	3	3	3	3	3
$6\,\text{tri}(n/3)$	0	2	4	6	4	2	0	0
$x[n]$	0	2	4	3	1	–1	–3	–3
$x^2[n]$	0	4	16	9	1	1	9	9

The sum of the squares is 49. Therefore $E_x = 49$.

2. Find the numerical value of the average signal power of a periodic signal $x(t)$ with period $T_0 = 3$, one period of which is described by

$$x(t) = \begin{cases} 2\,\text{rect}\left(\dfrac{t-1}{2}\right) - 5\,\text{rect}(t-1/2) & , \ 0 < t < 2 \\ -4 & , \ 2 < t < 3 \end{cases}.$$

$$P_x = \frac{1}{T_0}\int_{T_0} \left|x(t)\right|^2 dt = \text{average value of } x^2(t)$$

First fill in the table below with numbers.

t	$0 < t < 1$	$1 < t < 2$	$2 < t < 3$
$2\,\text{rect}\left(\frac{t-1}{2}\right)$	2	2	0
$5\,\text{rect}(t-1/2)$	5	0	0
$x(t)$	–3	2	–4
$x^2(t)$	9	4	16

Total area under $x^2(t)$ over one period is 29. Therefore the average value of $x^2(t)$ is 29/3 which is the same as the average signal power P_x.