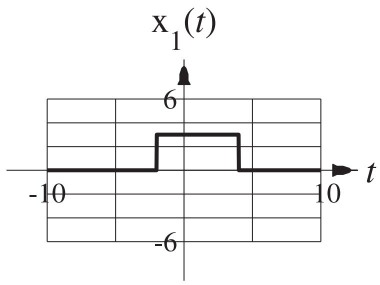
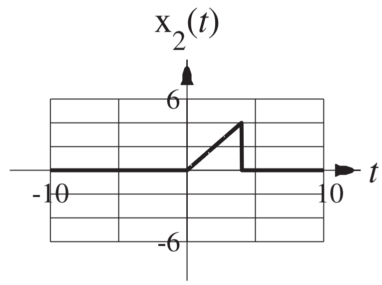
Solution of ECE 315 Test 1 F06

1. Let $x_1(t) = 3\,\text{rect}((t-1)/6)$ and $x_2(t) = \text{ramp}(t)[u(t) - u(t-4)]$.

   (a) Sketch them in the spaces provided below in the time range $-10 < t < 10$. Put scale numbers on the vertical axis so that actual numerical values could be read from the graph.





   (b) Find the numerical value of the signal energy of $x_1(t)$.

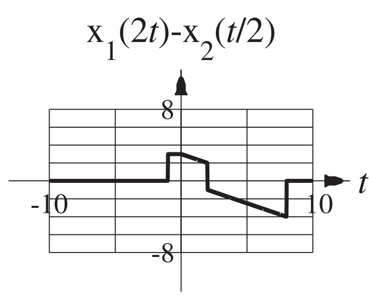
$$E_1 = \int_{-\infty}^{\infty} \left[3\,\text{rect}((t-1)/6)\right]^2 dt = 9\int_{-\infty}^{\infty} \text{rect}^2((t-1)/6)\,dt$$

$$E_1 = 9\int_{-2}^{4} dt = 54$$

   (c) Sketch $x(t) = x_1(2t) - x_2(t/2)$ below in the time range $-10 < t < 10$. Put scale numbers on the vertical axis so that actual numerical values could be read from the graph.

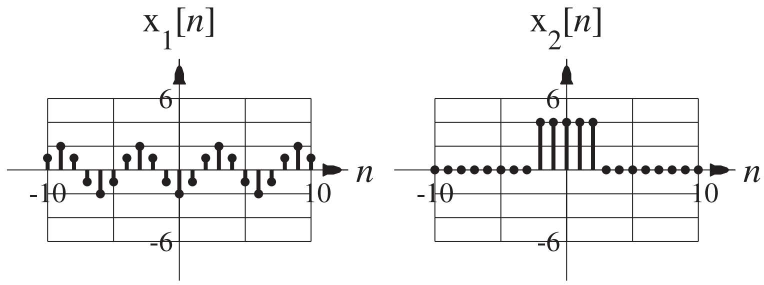
$x_1(2t) = 3\,\text{rect}((2t-1)/6)$ and $x_2(t/2) = \text{ramp}(t/2)[u(t/2) - u(t/2-4)]$

$x_1(2t) = 3\,\text{rect}((t-1/2)/3)$ and $x_2(t/2) = (1/2)\text{ramp}(t)[u(t) - u(t-8)]$



2. Let $x_1[n] = -2\cos(2\pi n/6)$ and $x_2[n] = 4(u[n+2] - u[n-3])$.

(a) Sketch them in the spaces provided below in the time range $-10 \le n \le 10$. Put scale numbers on the vertical axis so that actual numerical values could be read from the graph.



(b) Find the numerical value of the signal energy of $x_2[n]$.

$$E_2 = \sum_{n=-\infty}^{\infty} \left|4(u[n+2] - u[n-3])\right|^2 = 16\sum_{n=-2}^{2} 1 = 80$$

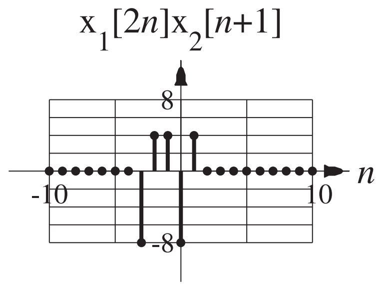
(c) Find the numerical value of the average signal power of $x_1[n]$.

$$P_1 = \frac{1}{6}\sum_{n=0}^{5} \left|-2\cos(2\pi n/6)\right|^2 = \frac{4}{6}\sum_{n=0}^{5}\cos^2(2\pi n/6)$$

$$P_1 = \frac{2}{3}\left[\cos^2(0) + \cos^2(\pi/3) + \cos^2(2\pi/3) + \cos^2(\pi) + \cos^2(4\pi/3) + \cos^2(5\pi/3)\right]$$

$$P_1 = \frac{2}{3}\left[1 + 0.25 + 0.25 + 1 + 0.25 + 0.25\right] = 2$$

(d) Sketch $x[n] = x_1[2n]x_2[n+1]$ below in the time range $-10 < n < 10$. Put scale numbers on the vertical axis so that actual numerical values could be read from the graph.



3. Find the numerical value of the fundamental period for these functions.

(a) $\mathrm{x}(t) = 3\cos(30\pi(t-2)) - 4\sin(45\pi t)$

$\mathrm{x}(t) = 3\cos(30\pi(t-2)) - 4\sin(45\pi t)$

Individual fundamental frequencies are 30/2 and 45/2. GCD of those is 15/2. Therefore overall fundamental frequency is 15/2 and overall fundamental period is 2/15 = 0.1333...

OR

Individual fundamental periods are 1/15 and 2/45. LCM of those is 2/15. Therefore overall fundamental period is 2/15.

(b) $\mathrm{x}[n] = 5\sin(33\pi n/12)$

In the form $\mathrm{x}[n] = A\cos\left(2\pi \frac{q}{N_0} n\right)$, if all common factors in q and N_0 have been cancelled, N_0 is the period.

$$\mathrm{x}[n] = 5\sin(33\pi n/12) = 5\sin\left(2\pi \frac{33}{24} n\right) = 5\sin\left(2\pi \frac{11}{8} n\right) \Rightarrow N_0 = 8$$

Alternate solution:

n	0	1	2	3	4	5	6	7	8	9	10	$\cdots$
$\mathrm{x}[n]$	0	3.536	–5	3.536	0	–3.536	5	–3.536	0	3.536	–5	$\cdots$

Recognize the repetition time as 8.

4. Let $\mathrm{x}[n] = \mathrm{tri}((n-2)/6)$ and let $\mathrm{x}_e[n]$ be its even part. What is the numerical value of $\mathrm{x}_e[1]$?

$$\mathrm{x}_e[n] = \frac{1}{2}\left[\mathrm{tri}((n-2)/6) + \mathrm{tri}((-n-2)/6)\right]$$

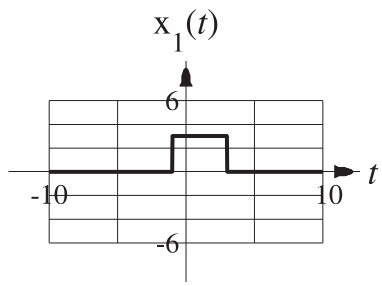
$$\mathrm{x}_e[1] = \frac{1}{2}\left[\mathrm{tri}((1-2)/6) + \mathrm{tri}((-1-2)/6)\right] = \frac{1}{2}\left[\mathrm{tri}(-1/6) + \mathrm{tri}(-1/2)\right]$$

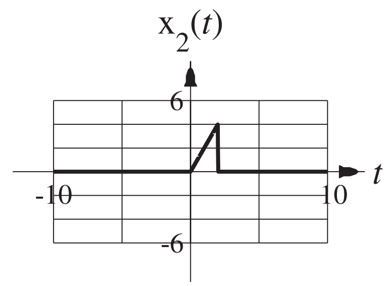
$$\mathrm{x}_e[1] = \frac{1}{2}(5/6 + 1/2) = 2/3$$

Solution of ECE 315 Test 1 F06

1. Let $x_1(t) = 3\,\mathrm{rect}((t-1)/4)$ and $x_2(t) = 2\,\mathrm{ramp}(t)[\mathrm{u}(t) - \mathrm{u}(t-2)]$.

(a) Sketch them in the spaces provided below in the time range $-10 < t < 10$. Put scale numbers on the vertical axis so that actual numerical values could be read from the graph.





(b) Find the numerical value of the signal energy of $x_1(t)$.

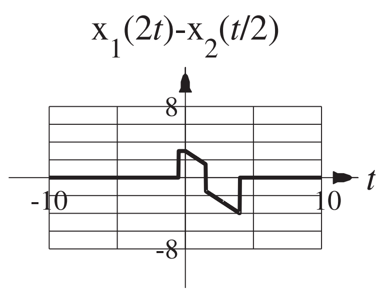
$$E_1 = \int_{-\infty}^{\infty} \left[3\,\mathrm{rect}((t-1)/4)\right]^2 dt = 9\int_{-\infty}^{\infty} \mathrm{rect}^2((t-1)/4)\,dt$$

$$E_1 = 9\int_{-1}^{3} dt = 36$$

(c) Sketch $x(t) = x_1(2t) - x_2(t/2)$ below in the time range $-10 < t < 10$. Put scale numbers on the vertical axis so that actual numerical values could be read from the graph.

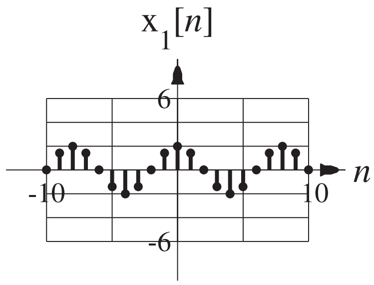
$$x_1(2t) = 3\,\mathrm{rect}((2t-1)/4) \text{ and } x_2(t/2) = 2\,\mathrm{ramp}(t/2)[\mathrm{u}(t/2) - \mathrm{u}(t/2-2)]$$

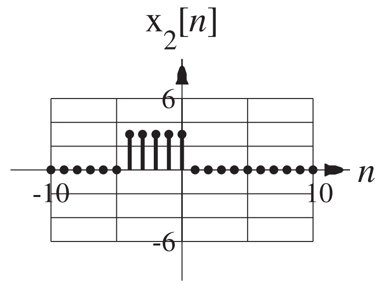
$$x_1(2t) = 3\,\mathrm{rect}((t-1/2)/2) \text{ and } x_2(t/2) = \mathrm{ramp}(t)[\mathrm{u}(t) - \mathrm{u}(t-4)]$$



2. Let $x_1[n] = 2\cos(2\pi n/8)$ and $x_2[n] = 3(u[n+4] - u[n-1])$.

(a) Sketch them in the spaces provided below in the time range $-10 \le n \le 10$. Put scale numbers on the vertical axis so that actual numerical values could be read from the graph.





(b) Find the numerical value of the signal energy of $x_2[n]$.

$$E_2 = \sum_{n=-\infty}^{\infty} \left|3(u[n+4] - u[n-1])\right|^2 = 9\sum_{n=-4}^{0} 1 = 45$$

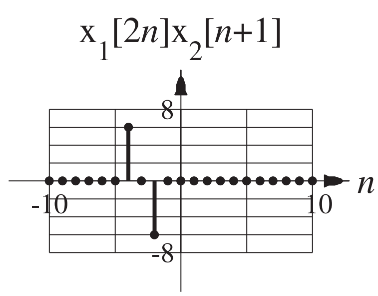
(c) Find the numerical value of the average signal power of $x_1[n]$.

$$P_1 = \frac{1}{8}\sum_{n=0}^{5}\left|2\cos(2\pi n/8)\right|^2 = \frac{4}{8}\sum_{n=0}^{5}\cos^2(2\pi n/8)$$

$$P_1 = \frac{1}{2}\begin{bmatrix} \cos^2(0) + \cos^2(\pi/4) + \cos^2(\pi/2) + \cos^2(3\pi/4) \\ +\cos^2(\pi) + \cos^2(5\pi/4) + \cos^2(3\pi/2) + \cos^2(7\pi/2) \end{bmatrix}$$

$$P_1 = \frac{1}{2}\left[1 + 0.5 + 0 + 0.5 + 1 + 0.5 + 0 + 0.5\right] = 2$$

(d) Sketch $x[n] = x_1[2n]x_2[n+1]$ below in the time range $-10 < n < 10$. Put scale numbers on the vertical axis so that actual numerical values could be read from the graph.



3. Find the numerical value of the fundamental period for these functions.

(a) $\mathrm{x}(t) = 3\cos(40\pi(t-2)) - 4\sin(50\pi t)$

$$\mathrm{x}(t) = 3\cos(40\pi(t-2)) - 4\sin(50\pi t)$$

Individual fundamental frequencies are 20 and 25. GCD of those is 5. Therefore overall fundamental frequency is 5 and overall fundamental period is 1/5

OR

Individual fundamental periods are 1/20 and 1/25. LCM of those is 1/5. Therefore overall fundamental period is 1/5.

(b) $\mathrm{x}[n] = 5\sin(33\pi n/6)$

In the form $\mathrm{x}[n] = A\cos\left(2\pi \frac{q}{N_0} n\right)$, if all common factors in q and N_0 have been cancelled, N_0 is the period.

$$\mathrm{x}[n] = 5\sin(33\pi n/6) = 5\sin\left(2\pi \frac{33}{12} n\right) = 5\sin\left(2\pi \frac{11}{4} n\right) \Rightarrow N_0 = 4$$

Alternate solution:

n	0	1	2	3	4	5	6	7	8	9	10	$\cdots$
$\mathrm{x}[n]$	0	−5	0	5	0	−5	0	5	0	−5	0	$\cdots$

Recognize the repetition time as 4.

4. Let $\mathrm{x}[n] = \mathrm{tri}((n-2)/5)$ and let $\mathrm{x}_o[n]$ be its odd part. What is the numerical value of $\mathrm{x}_o[1]$?

$$\mathrm{x}_o[n] = \frac{1}{2}\left[\mathrm{tri}((n-2)/5) - \mathrm{tri}((-n-2)/5)\right]$$

$$\mathrm{x}_o[1] = \frac{1}{2}\left[\mathrm{tri}((1-2)/5) - \mathrm{tri}((-1-2)/5)\right] = \frac{1}{2}\left[\mathrm{tri}(-1/5) - \mathrm{tri}(-3/5)\right]$$

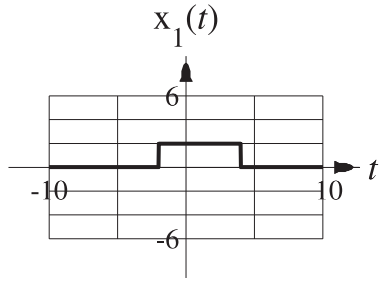
$$\mathrm{x}_o[1] = \frac{1}{2}(4/5 - 2/5) = 1/5$$

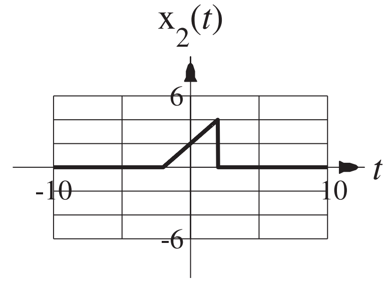
Solution of ECE 315 Test 1 F06

1. Let

$$x_1(t) = 2\,\text{rect}((t-1)/6) \text{ and } x_2(t) = \text{ramp}(t+2)[u(t+2) - u(t-2)].$$

(a) Sketch them in the spaces provided below in the time range $-10 < t < 10$. Put scale numbers on the vertical axis so that actual numerical values could be read from the graph.





(b) Find the numerical value of the signal energy of $x_1(t)$.

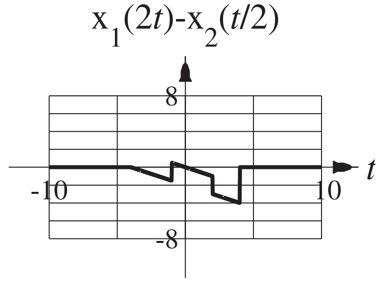
$$E_1 = \int_{-\infty}^{\infty} [2\,\text{rect}((t-1)/6)]^2\, dt = 4\int_{-\infty}^{\infty} \text{rect}^2((t-1)/6)\, dt$$

$$E_1 = 4\int_{-2}^{4} dt = 24$$

(c) Sketch $x(t) = x_1(2t) - x_2(t/2)$ below in the time range $-10 < t < 10$. Put scale numbers on the vertical axis so that actual numerical values could be read from the graph.

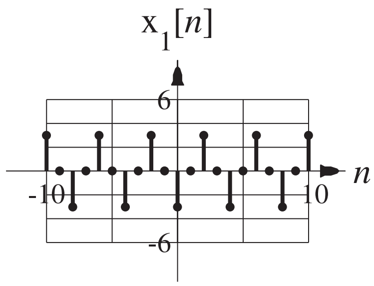
$$x_1(2t) = 2\,\text{rect}((2t-1)/6) \text{ and } x_2(t/2) = \text{ramp}(t/2+2)[u(t/2+2) - u(t/2-2)]$$

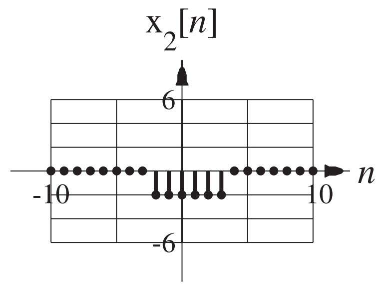
$$x_1(2t) = 2\,\text{rect}((t-1/2)/3) \text{ and } x_2(t/2) = (1/2)\text{ramp}(t+4)[u(t+4) - u(t-4)]$$



2. Let $x_1[n] = -3\cos(2\pi n/4)$ and $x_2[n] = -2(u[n+2] - u[n-4])$.

(a) Sketch them in the spaces provided below in the time range $-10 \le n \le 10$. Put scale numbers on the vertical axis so that actual numerical values could be read from the graph.





(b) Find the numerical value of the signal energy of $x_2[n]$.

$$E_2 = \sum_{n=-\infty}^{\infty} \left|-2(u[n+2] - u[n-4])\right|^2 = 4\sum_{n=-2}^{3} 1 = 24$$

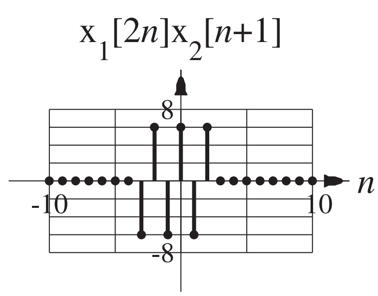
(c) Find the numerical value of the average signal power of $x_1[n]$.

$$P_1 = \frac{1}{4}\sum_{n=0}^{5} \left|-3\cos(2\pi n/4)\right|^2 = \frac{9}{4}\sum_{n=0}^{5} \cos^2(2\pi n/4)$$

$$P_1 = \frac{9}{4}\left[\cos^2(0) + \cos^2(\pi/2) + \cos^2(\pi) + \cos^2(3\pi/2)\right]$$

$$P_1 = \frac{9}{4}\left[1+0+1+0\right] = 9/2 = 4.5$$

(d) Sketch $x[n] = x_1[2n]x_2[n+1]$ below in the time range $-10 < n < 10$. Put scale numbers on the vertical axis so that actual numerical values could be read from the graph.



3. Find the numerical value of the fundamental period for these functions.

(a) $\mathrm{x}(t) = 3\cos(18\pi(t-2)) - 4\sin(24\pi t)$

$\mathrm{x}(t) = 3\cos(18\pi(t-2)) - 4\sin(24\pi t)$

Individual fundamental frequencies are 9 and 12. GCD of those is 3.
Therefore overall fundamental frequency is 3 and overall fundamental period is 1/3 = 0.3333...

OR

Individual fundamental periods are 1/9 and 1/12. LCM of those is 1/3.
Therefore overall fundamental period is 1/3.

(b) $\mathrm{x}[n] = 5\sin(33\pi n / 9)$

In the form $\mathrm{x}[n] = A\cos\left(2\pi \frac{q}{N_0} n\right)$, if all common factors in q and N_0 have been cancelled, N_0 is the period.

$$\mathrm{x}[n] = 5\sin(33\pi n / 9) = 5\sin\left(2\pi \frac{33}{18} n\right) = 5\sin\left(2\pi \frac{11}{6} n\right) \Rightarrow N_0 = 6$$

Alternate solution:

n	0	1	2	3	4	5	6	7	8	9	10	$\cdots$
$\mathrm{x}[n]$	0	−4.33	−4.33	0	4.33	4.33	0	−4.33	−4.33	0	4.33	$\cdots$

Recognize the repetition time as 6.

4. Let $\mathrm{x}[n] = \mathrm{tri}((n-2)/6)$ and let $\mathrm{x}_e[n]$ be its even part. What is the numerical value of $\mathrm{x}_e[1]$?

$$\mathrm{x}_e[n] = \frac{1}{2}\left[\mathrm{tri}((n-2)/6) + \mathrm{tri}((-n-2)/6)\right]$$

$$\mathrm{x}_e[1] = \frac{1}{2}\left[\mathrm{tri}((1-2)/6) + \mathrm{tri}((-1-2)/6)\right] = \frac{1}{2}\left[\mathrm{tri}(-1/6) + \mathrm{tri}(-1/2)\right]$$

$$\mathrm{x}_e[1] = \frac{1}{2}(5/6 + 1/2) = 2/3$$