

## Solution of ECE 315 Test 1 F07

1. A signal  $x(t)$  is described by  $x(t) = \begin{cases} -5t, & 0 \leq t < 3 \\ 0, & \text{otherwise} \end{cases}$ . Find the numerical value of its signal energy.

$E_x =$  \_\_\_\_\_

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_0^3 |-5t|^2 dt = 25 \int_0^3 t^2 dt = 25 \left[ \frac{t^3}{3} \right]_0^3 = 25(9) = 225$$

2. A periodic signal  $x(t)$  is described over one period by  $x(t) = \begin{cases} 3, & 0 \leq t < 2 \\ -1, & 2 \leq t < 5 \end{cases}$ .

- (a) Find the numerical value of its average signal power  $P_x$ .  $P_x =$  \_\_\_\_\_

$$P_x = \frac{1}{T} \int_T |x(t)|^2 dt = \frac{1}{5} \int_0^5 |x(t)|^2 dt = \frac{1}{5} \left[ \int_0^2 3^2 dt + \int_2^5 (-1)^2 dt \right]$$

$$P_x = \frac{1}{5} \left[ 9 \int_0^2 dt + \int_2^5 dt \right] = \frac{1}{5} (9 \times 2 + 5 - 2) = 21/5 = 4.2$$

- (b) If  $y(t) = x(3t)$  what is the numerical average signal power of  $y(t)$ ,  $P_y$ ?

$P_y =$  \_\_\_\_\_

The signal power does not change when a periodic signal is compressed or expanded in time. So the answer is the same, 4.2.

- (c) If  $z(t) = 3x(t-2)$  what is the numerical average signal power of  $z(t)$ ,  $P_z$ ?

$P_z =$  \_\_\_\_\_

Every point on the signal is now 3 times greater in amplitude. The shift does not affect the average signal power. Therefore the average signal power is  $3^2$  times what it was before or  $9 \times 4.2 = 37.8$ .

## Solution of ECE 315 Test 1 F07

1. A signal  $x(t)$  is described by  $x(t) = \begin{cases} -2t, & 0 \leq t < 4 \\ 0, & \text{otherwise} \end{cases}$ . Find the numerical value of its signal energy.

$$E_x = \underline{\hspace{4cm}}$$

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_0^4 |-2t|^2 dt = 4 \int_0^4 t^2 dt = 4 \left[ \frac{t^3}{3} \right]_0^4 = 256/3 = 85.333$$

2. A periodic signal  $x(t)$  is described over one period by  $x(t) = \begin{cases} 2, & 0 \leq t < 5 \\ -3, & 5 \leq t < 9 \end{cases}$ .

- (a) Find the numerical value of its average signal power  $P_x$ .  $P_x = \underline{\hspace{4cm}}$

$$P_x = \frac{1}{T} \int_T |x(t)|^2 dt = \frac{1}{9} \int_0^9 |x(t)|^2 dt = \frac{1}{9} \left[ \int_0^5 |2|^2 dt + \int_5^9 |-3|^2 dt \right]$$

$$P_x = \frac{1}{9} \left[ 4 \int_0^5 dt + 9 \int_5^9 dt \right] = \frac{1}{9} (4 \times 5 + 9 \times (9 - 5)) = 56/9 = 6.222$$

- (b) If  $y(t) = x(3t)$  what is the numerical average signal power of  $y(t)$ ,  $P_y$ ?

$$P_y = \underline{\hspace{4cm}}$$

The signal power does not change when a periodic signal is compressed or expanded in time. So the answer is the same, 6.222.

- (c) If  $z(t) = 2x(t-2)$  what is the numerical average signal power of  $z(t)$ ,  $P_z$ ?

$$P_z = \underline{\hspace{4cm}}$$

Every point on the signal is now 2 times greater in amplitude. The shift does not affect the average signal power. Therefore the average signal power is  $2^2$  times what it was before or  $4 \times 6.222 = 24.889$ .

## Solution of ECE 315 Test 1 F07

1. A signal  $x(t)$  is described by  $x(t) = \begin{cases} -4t, & 0 \leq t < 10 \\ 0, & \text{otherwise} \end{cases}$ . Find the numerical value of its signal energy.

$$E_x = \underline{\hspace{4cm}}$$

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_0^{10} |-4t|^2 dt = 16 \int_0^{10} t^2 dt = 16 \left[ \frac{t^3}{3} \right]_0^{10} = 16000 / 3 = 5333.3$$

2. A periodic signal  $x(t)$  is described over one period by  $x(t) = \begin{cases} 7, & 0 \leq t < 1 \\ -2, & 1 \leq t < 6 \end{cases}$ .

- (a) Find the numerical value of its average signal power  $P_x$ .  $P_x = \underline{\hspace{4cm}}$

$$P_x = \frac{1}{T} \int_T |x(t)|^2 dt = \frac{1}{6} \int_0^6 |x(t)|^2 dt = \frac{1}{6} \left[ \int_0^1 |7|^2 dt + \int_1^6 |-2|^2 dt \right]$$

$$P_x = \frac{1}{6} \left[ 49 \int_0^1 dt + 4 \int_1^6 dt \right] = \frac{1}{6} (49 + 4 \times (6-1)) = 69 / 6 = 11.5$$

- (b) If  $y(t) = x(3t)$  what is the numerical average signal power of  $y(t)$ ,  $P_y$ ?

$$P_y = \underline{\hspace{4cm}}$$

The signal power does not change when a periodic signal is compressed or expanded in time. So the answer is the same, 11.5.

- (c) If  $z(t) = 4x(t-2)$  what is the numerical average signal power of  $z(t)$ ,  $P_z$ ?

$$P_z = \underline{\hspace{4cm}}$$

Every point on the signal is now 4 times greater in amplitude. The shift does not affect the average signal power. Therefore the average signal power is  $4^2$  times what it was before or  $16 \times 11.5 = 184$ .