Solution of ECE 315 Test 2 F08

1. Provide a numerical answer. (Remember a specification like $n_0 \le n < n_1$ means values of *n* from n_0 to $n_1 - 1$, inclusive. The range does not include n_1.)

   (a) Signal energy of $\mathrm{x}[n] = 4\,\mathrm{rect}_3[n]$

$$E_x = \sum_{n=-\infty}^{\infty} \left|\mathrm{x}[n]\right|^2 = \sum_{n=-\infty}^{\infty} \left|4\,\mathrm{rect}_3[n]\right|^2 = 16\sum_{n=-3}^{3} 1 = 112$$

   (b) Average signal power of a periodic signal described over one period by $\mathrm{x}[n] = 2n \;\;,\;\; -2 \le n < 2$.

$$P_x = \frac{1}{4}\sum_{n=\langle 4\rangle} \left|\mathrm{x}[n]\right|^2 = \frac{1}{4}\sum_{n=-2}^{1} (2n)^2 = \sum_{n=-2}^{1} n^2 = 4+1+0+1 = 6$$

   (c) Average signal power of $\mathrm{x}[n] = -5 + 3\sin(2\pi n/4)$

$$P_x = \frac{1}{4}\sum_{n=\langle 4\rangle} \left|\mathrm{x}[n]\right|^2 = \frac{1}{4}\sum_{n=\langle 4\rangle} \left|-5 + 3\sin(2\pi n/4)\right|^2 = \frac{1}{4}\sum_{n=0}^{3}\left[-5 + 3\sin(2\pi n/4)\right]^2$$

$$P_x = \frac{1}{4}\left[(-5)^2 + (-2)^2 + (-5)^2 + (-8)^2\right] = 118/4 = 59/2 = 29.5$$

2. If $\mathrm{x}[n] = \begin{cases} 6n &, -2 \le n < 2 \\ 0 &, \text{otherwise} \end{cases}$ and $\mathrm{y}[n] = \mathrm{x}[2n]$, find the numerical signal energy of $\mathrm{y}[n]$.

$$\mathrm{y}[n] = \mathrm{x}[2n] = \begin{cases} 12n &, -2 \le 2n < 2 \\ 0 &, \text{otherwise} \end{cases} = \begin{cases} 12n &, -1 \le n < 1 \\ 0 &, \text{otherwise} \end{cases}$$

$$E_y = \sum_{n=-\infty}^{\infty} \left|\mathrm{y}[n]\right|^2 = \sum_{n=-1}^{0} \left|12n\right|^2 = 144\sum_{n=-1}^{0} n^2 = 144(1+0) = 144$$

Solution of ECE 315 Test 2 F08

1. Provide a numerical answer. (Remember a specification like $n_0 \le n < n_1$ means values of n from n_0 to $n_1 - 1$, inclusive. The range does not include n_1.)

   (a) Signal energy of $\mathrm{x}[n] = 3\mathrm{rect}_3[n]$

$$E_x = \sum_{n=-\infty}^{\infty} \left|\mathrm{x}[n]\right|^2 = \sum_{n=-\infty}^{\infty} \left|3\mathrm{rect}_3[n]\right|^2 = 9\sum_{n=-3}^{3} 1 = 63$$

   (b) Average signal power of a periodic signal described over one period by $\mathrm{x}[n] = 5n \ , \ -2 \le n < 2$.

$$P_x = \frac{1}{4}\sum_{n=\langle 4\rangle} \left|\mathrm{x}[n]\right|^2 = \frac{1}{4}\sum_{n=-2}^{1}(5n)^2 = (25/4)\sum_{n=-2}^{1} n^2 = (25/4)(4+1+0+1) = 37.5$$

   (c) Average signal power of $\mathrm{x}[n] = -3 + 4\sin(2\pi n/4)$

$$P_x = \frac{1}{4}\sum_{n=\langle 4\rangle} \left|\mathrm{x}[n]\right|^2 = \frac{1}{4}\sum_{n=\langle 4\rangle} \left|-3 + 4\sin(2\pi n/4)\right|^2 = \frac{1}{4}\sum_{n=0}^{3}\left[-3 + 4\sin(2\pi n/4)\right]^2$$

$$P_x = \frac{1}{4}\left[(-3)^2 + 1^2 + (-3)^2 + (-7)^2\right] = 68/4 = 17$$

2. If $\mathrm{x}[n] = \begin{cases} 4n \ , & -2 \le n < 2 \\ 0 \ , & \text{otherwise} \end{cases}$ and $\mathrm{y}[n] = \mathrm{x}[2n]$, find the numerical signal energy of $\mathrm{y}[n]$.

$$\mathrm{y}[n] = \mathrm{x}[2n] = \begin{cases} 8n \ , & -2 \le 2n < 2 \\ 0 \ , & \text{otherwise} \end{cases} = \begin{cases} 8n \ , & -1 \le n < 1 \\ 0 \ , & \text{otherwise} \end{cases}$$

$$E_y = \sum_{n=-\infty}^{\infty} \left|\mathrm{y}[n]\right|^2 = \sum_{n=-1}^{0} |8n|^2 = 64\sum_{n=-1}^{0} n^2 = 64(1+0) = 64$$

Solution of ECE 315 Test 2 F08

1. Provide a numerical answer. (Remember a specification like $n_0 \le n < n_1$ means values of *n* from n_0 to $n_1 - 1$, inclusive. The range does not include n_1.)

   (a) Signal energy of $\mathrm{x}[n] = 7\,\mathrm{rect}_3[n]$

$$E_x = \sum_{n=-\infty}^{\infty} \left|\mathrm{x}[n]\right|^2 = \sum_{n=-\infty}^{\infty} \left|7\,\mathrm{rect}_3[n]\right|^2 = 49\sum_{n=-3}^{3} 1 = 343$$

   (b) (Average signal power of a periodic signal described over one period by $\mathrm{x}[n] = 9n \;,\; -2 \le n < 2$.

$$P_x = \frac{1}{4}\sum_{n=\langle 4\rangle} \left|\mathrm{x}[n]\right|^2 = \frac{1}{4}\sum_{n=-2}^{1} (9n)^2 = (81/4)\sum_{n=-2}^{1} n^2 = (81/4)(4+1+0+1) = 121.5$$

   (c) Average signal power of $\mathrm{x}[n] = -2 + 6\sin(2\pi n/4)$

$$P_x = \frac{1}{4}\sum_{n=\langle 4\rangle} \left|\mathrm{x}[n]\right|^2 = \frac{1}{4}\sum_{n=\langle 4\rangle} \left|-2+6\sin(2\pi n/4)\right|^2 = \frac{1}{4}\sum_{n=0}^{3}\left[-2+6\sin(2\pi n/4)\right]^2$$

$$P_x = \frac{1}{4}\left[(-2)^2 + (4)^2 + (-2)^2 + (-8)^2\right] = 88/4 = 22$$

2. If $\mathrm{x}[n] = \begin{cases} 3n \;,\; -2 \le n < 2 \\ 0 \;,\; \text{otherwise} \end{cases}$ and $\mathrm{y}[n] = \mathrm{x}[2n]$, find the numerical signal energy of $\mathrm{y}[n]$.

$$\mathrm{y}[n] = \mathrm{x}[2n] = \begin{cases} 6n \;,\; -2 \le 2n < 2 \\ 0 \;,\; \text{otherwise} \end{cases} = \begin{cases} 6n \;,\; -1 \le n < 1 \\ 0 \;,\; \text{otherwise} \end{cases}$$

$$E_y = \sum_{n=-\infty}^{\infty} \left|\mathrm{y}[n]\right|^2 = \sum_{n=-1}^{0} |6n|^2 = 36\sum_{n=-1}^{0} n^2 = 36(1+0) = 36$$