Solution of EECS 315 Test 1 F10

1. Find the numerical magnitude and phase in radians of this function. (If the phase is undefined just write "undefined").

   $e^{(15+j4)f}$ at $f = -0.2$ $\qquad$ $0.04979\angle -0.8$

2. Find the numerical fundamental periods of these signals.

   (a) $\mathrm{x}[n] = 13\delta_{16}[n] - 11\delta_{20}[n]$

   Fundamental period is LCM of 16 and 20 which is 80

   (b) $\mathrm{x}[n] = 4\sin(18\pi n/24) - 3\cos(10\pi n/16)$

   $\mathrm{x}[n] = 4\sin(2\pi n(9/24)) - 3\cos(2\pi n(5/16)) = 4\sin(2\pi n(3/8)) - 3\cos(2\pi n(5/16))$

   Fundamental period is LCM of 8 and 16 which is 16.

(a) Is $\mathrm{x}(t)$ even, odd or neither? Even Odd Neither

Neither

(b) Is $\mathrm{y}(t)$ even, odd or neither? Even Odd Neither

Even

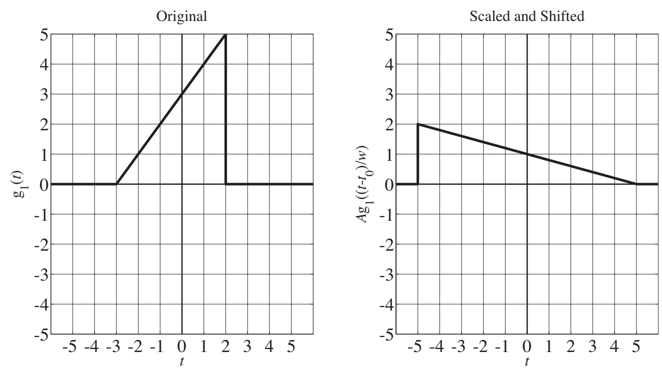
4. If $\mathrm{x}(t) = 5\delta(4(t-2))$, find the numerical value of $\int_{-3}^{8} \mathrm{x}(t)\,dt$.

$$\int_{-3}^{8} \mathrm{x}(t)\,dt = \int_{-3}^{8} 5\delta(4(t-2))\,dt = (5/4)\int_{-3}^{8} \delta(t-2)\,dt = 5/4$$

5. If $\mathrm{x}[n] = 5\delta_{12}[4(n-2)]$, find the numerical value of $\sum_{n=-3}^{8} \mathrm{x}[n]$.

$$\sum_{n=-3}^{8} \mathrm{x}[n] = \sum_{n=-3}^{8} 5\delta_{12}[4(n-2)] = 5\sum_{n=-3}^{8} \delta_{12}[4(n-2)] = 5(0+0+1+0+0+1+0+0+1+0+0+1) = 20$$

6. The amplitude scaling factor is A and the time scaling and shifting between the original signal and the scaled and shifted signal is in the form $t \to \frac{t-t_0}{w}$. Find numerical values of A, t_0 and w.



$A = 0.4$, $t_0 = -1$, $w = -2$

7. If $x(t) = 3t^2 + 9t + 4$ and $x_e(t)$ is the even part of $x(t)$, find the numerical value of $x_e(5)$.

$$x_e(t) = 3t^2 + 4 \Rightarrow x_e(5) = 79$$

8. Find the numerical signal energy of $x[n] = 0.8^{2n}\left(u[n+2] - u[n-1]\right)$.

$$E_x = \sum_{n=-\infty}^{\infty} \left|0.8^{2n}\left(u[n+2] - u[n-1]\right)\right|^2 = \sum_{n=-2}^{0} 0.8^{4n} = 0.8^{-8} + 0.8^{-4} + 0.8^{0} = 9.4019$$

9. If one period of $x(t)$ is described by $x(t) = \text{rect}(2t) - 3$, $-1 < t < 1$, find the numerical average signal power of $x(t)$.

$$P_x = \frac{1}{T}\int_T |x(t)|^2\, dt = \frac{1}{2}\int_{-1}^{1} \Big|\underbrace{\text{rect}(2t) - 3}_{\text{even}}\Big|^2 dt = \int_0^1 \left[\text{rect}^2(2t) + 9 - 6\,\text{rect}(2t)\right] dt$$

$$P_x = \int_0^{1/4} dt + 9\int_0^1 dt - 6\int_0^{1/4} dt = 1/4 + 9 - 6/4 = 31/4 = 7.75$$

Solution of EECS 315 Test 1 F10

1. Find the numerical magnitude and phase in radians of this function. (If the phase is undefined just write "undefined").

$e^{(12+j3)f}$ at $f=-0.2$ $\quad 0.09072\angle -0.6$

2. Find the numerical fundamental periods of these signals.

(a) $\mathrm{x}[n]=13\delta_{32}[n]-11\delta_{40}[n]$

Fundamental period is LCM of 32 and 40 which is 160

(b) $\mathrm{x}[n]=4\sin(18\pi n/12)-3\cos(10\pi n/8)$

$\mathrm{x}[n]=4\sin(2\pi n(9/12))-3\cos(2\pi n(5/8))=4\sin(2\pi n(3/4))-3\cos(2\pi n(5/8))$

Fundamental period is LCM of 4 and 8 which is 8.

3. A continuous-time function is defined by $\mathrm{x}(t)=4\,\mathrm{u}(t)-t$ and $\mathrm{y}(t)$ is the generalized derivative of $\mathrm{x}(t)$.

(a) (1 pt) Is $\mathrm{x}(t)$ even, odd or neither?

Neither

(b) Is $\mathrm{y}(t)$ even, odd or neither?

Even

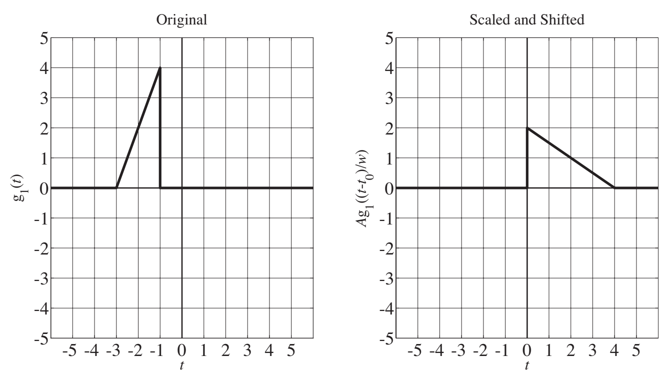
4. If $x(t) = 9\delta(4(t-2))$, find the numerical value of $\int_{-3}^{8} x(t)\,dt$.

$$\int_{-3}^{8} x(t)\,dt = \int_{-3}^{8} 9\delta(4(t-2))\,dt = (9/4)\int_{-3}^{8} \delta(t-2)\,dt = 9/4$$

5. If $x[n] = 3\delta_9[3(n-2)]$, find the numerical value of $\sum_{n=-3}^{8} x[n]$.

$$\sum_{n=-3}^{8} x[n] = \sum_{n=-3}^{8} 3\delta_9[3(n-2)] = 3\sum_{n=-3}^{8} \delta_9[3(n-2)] = 3(0+0+1+0+0+1+0+0+1+0+0+1) = 12$$

6. The amplitude scaling factor is A and the time scaling and shifting between the original signal and the scaled and shifted signal is in the form $t \rightarrow \frac{t-t_0}{w}$. Find numerical values of A, t_0 and w.



A = 0.5 , t_0 = -2 , w = -2

7. If $\mathrm{x}(t) = 5t^2 - 4t + 2$ and $\mathrm{x}_e(t)$ is the even part of $\mathrm{x}(t)$, find the numerical value of $\mathrm{x}_e(5)$.

$$\mathrm{x}_e(t) = 5t^2 + 2 \Rightarrow \mathrm{x}_e(5) = 127$$

8. Find the numerical signal energy of $\mathrm{x}[n] = 0.9^{2n}(\mathrm{u}[n+3] - \mathrm{u}[n])$.

$$E_{\mathrm{x}} = \sum_{n=-\infty}^{\infty} \left|0.9^{2n}(\mathrm{u}[n+3] - \mathrm{u}[n])\right|^2 = \sum_{n=-3}^{-1} 0.9^{4n} = 0.9^{-12} + 0.9^{-8} + 0.9^{-4} = 7.3879$$

9. If one period of $\mathrm{x}(t)$ is described by $\mathrm{x}(t) = \mathrm{rect}(2t) - 4$, $-1 < t < 1$, find the numerical average signal power of $\mathrm{x}(t)$.

$$P_{\mathrm{x}} = \frac{1}{T}\int_T |\mathrm{x}(t)|^2\,dt = \frac{1}{2}\int_{-1}^{1}\left|\underbrace{\mathrm{rect}(2t) - 4}_{\text{even}}\right|^2 dt = \int_0^1 \left[\mathrm{rect}^2(2t) + 16 - 8\,\mathrm{rect}(2t)\right]dt$$

$$P_{\mathrm{x}} = \int_0^{1/4} dt + 16\int_0^1 dt - 8\int_0^{1/4} dt = 1/4 + 16 - 2 = 57/4 = 14.25$$

Solution of EECS 315 Test 1 F10

1. Find the numerical magnitude and phase in radians of this function. (If the phase is undefined just write "undefined").

$e^{(15+j4)f}$ at $f = -0.2$ $\quad$ $0.36788\angle -0.6$

2. Find the numerical fundamental periods of these signals.

(a) $x[n] = 13\delta_{48}[n] - 11\delta_{60}[n]$

Fundamental period is LCM of 48 and 60 which is 240

(c) $x[n] = 4\sin(18\pi n/6) - 3\cos(10\pi n/4)$

$$x[n] = 4\sin(2\pi n(9/6)) - 3\cos(2\pi n(5/4)) = 4\sin(2\pi n(3/2)) - 3\cos(2\pi n(5/4))$$

Fundamental period is LCM of 2 and 4 which is 4.

3. A continuous-time function is defined by $x(t) = t - 2u(t)$ and $y(t)$ is the generalized derivative of $x(t)$.

(a) (1 pt) Is $x(t)$ even, odd or neither?

Neither

(b) Is $y(t)$ even, odd or neither?

Even

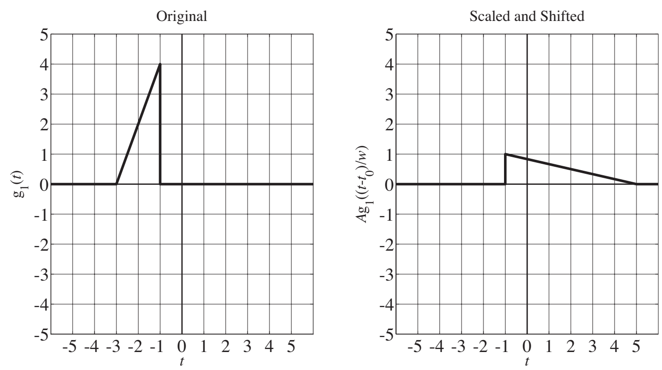
4. If $\mathrm{x}(t) = 13\delta(7(t-2))$, find the numerical value of $\int_{-3}^{8} \mathrm{x}(t)\,dt$.

$$\int_{-3}^{8} \mathrm{x}(t)\,dt = \int_{-3}^{8} 13\delta(7(t-2))\,dt = (13/7)\int_{-3}^{8} \delta(t-2)\,dt = 13/7 \cong 1.8571$$

5. If $\mathrm{x}[n] = 8\delta_9[3(n-2)]$, find the numerical value of $\sum_{n=-3}^{8} \mathrm{x}[n]$.

$$\sum_{n=-3}^{8} \mathrm{x}[n] = \sum_{n=-3}^{8} 8\delta_9[3(n-2)] = 8\sum_{n=-3}^{8} \delta_9[3(n-2)] = 8(0+0+1+0+0+1+0+0+1+0+0+1) = 32$$

6. The amplitude scaling factor is A and the time scaling and shifting between the original signal and the scaled and shifted signal is in the form $t \to \dfrac{t-t_0}{w}$. Find numerical values of A, t_0 and w.



A = 0.25 , t_0 = -4 , w = -3

7. If $\mathrm{x}(t) = -2t^2 + 11t + 1$ and $\mathrm{x}_e(t)$ is the even part of $\mathrm{x}(t)$, find the numerical value of $\mathrm{x}_e(5)$.

$$\mathrm{x}_e(t) = -2t^2 + 1 \Rightarrow \mathrm{x}_e(5) = -49$$

8. Find the numerical signal energy of $\mathrm{x}[n] = 0.7^{2n}\left(\mathrm{u}[n+4] - \mathrm{u}[n+1]\right)$.

$$E_{\mathrm{x}} = \sum_{n=-\infty}^{\infty} \left|0.7^{2n}\left(\mathrm{u}[n+4] - \mathrm{u}[n+1]\right)\right|^2 = \sum_{n=-4}^{-2} 0.7^{4n} = 0.7^{-16} + 0.7^{-12} + 0.7^{-8} = 390.5$$

9. If one period of $\mathrm{x}(t)$ is described by $\mathrm{x}(t) = \mathrm{rect}(t) - 1$, $-1 < t < 1$, find the numerical average signal power of $\mathrm{x}(t)$.

$$P_{\mathrm{x}} = \frac{1}{T}\int_T \left|\mathrm{x}(t)\right|^2 dt = \frac{1}{2}\int_{-1}^{1} \left|\underbrace{\mathrm{rect}(t) - 1}_{\text{even}}\right|^2 dt = \int_0^1 \left[\mathrm{rect}^2(t) + 1 - 2\,\mathrm{rect}(t)\right] dt$$

$$P_{\mathrm{x}} = \int_0^{1/2} dt + \int_0^1 dt - 2\int_0^{1/2} dt = 1/2 + 1 - 1 = 1/2$$