Solution of EECS 315 Test 5 F13

1. The impulse response of a digital filter is $\mathrm{x}[n]=3.2(0.8)^n\,\mathrm{u}[n]$. Find the numerical signal energy of $\mathrm{x}[n]$, E_x.

$$E_x=\sum_{n=-\infty}^{\infty}\left|\mathrm{x}[n]\right|^2=3.2^2\sum_{n=0}^{\infty}\left(0.8^n\right)^2=10.24\sum_{n=0}^{\infty}\left(0.8^2\right)^n$$

$$E_x=10.24\sum_{n=0}^{\infty}0.64^n=10.24\frac{1}{1-0.64}=28.444$$

2. Find the numerical average signal power of $\mathrm{x}[n]=11\delta_4[2n]-15\delta_5[3n]$.

Using the definition of the discrete-time periodic impulse, $\delta_N[n]=\sum_{m=-\infty}^{\infty}\delta[n-mN]$,

$$11\delta_4[2n]=11\sum_{m=-\infty}^{\infty}\delta[2n-4m]\ ,\ \text{Impulses occur when } 2n-4m=0\Rightarrow n=2m\,.$$

Since m only takes on integer values and $n=2m$ the impulses occur when n is an integer multiple of 2.

$$15\delta_5[3n]=15\sum_{m=-\infty}^{\infty}\delta[3n-5m]\ ,\ \text{Impulses occur when } 3n-5m=0\Rightarrow n=5m/3\,.$$

Since n and m must be integers, the impulses occur when $5m/3$ is an integer or when m is an integer multiple of 3 making n an integer multiple of 5. So the fundamental period of $\mathrm{x}[n]$ is the LCM of 2 and 5 which is 10.

$$P_x=\frac{1}{10}\sum_{n=0}^{9}\left|\mathrm{x}[n]\right|^2=\frac{1}{10}\left[(-4)^2+0^2+11^2+0^2+11^2+(-15)^2+11^2+0^2+11^2+0^2\right]$$

$$P_x=\frac{16+121\times4+225}{10}=72.5$$

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1. The impulse response of a digital filter is $\mathrm{x}[n]=3.2(0.6)^n\,\mathrm{u}[n]$. Find the numerical signal energy of $\mathrm{x}[n]$, E_x.

$$E_x=\sum_{n=-\infty}^{\infty}\left|\mathrm{x}[n]\right|^2=3.2^2\sum_{n=0}^{\infty}\left(0.6^n\right)^2=10.24\sum_{n=0}^{\infty}\left(0.6^2\right)^n$$

$$E_x=10.24\sum_{n=0}^{\infty}0.36^n=10.24\frac{1}{1-0.36}=16$$

2. Find the numerical average signal power of $\mathrm{x}[n]=11\delta_4[2n]-18\delta_5[3n]$.

Using the definition of the discrete-time periodic impulse, $\delta_N[n]=\sum_{m=-\infty}^{\infty}\delta[n-mN]$,

$$11\delta_4[2n]=11\sum_{m=-\infty}^{\infty}\delta[2n-4m]$$, Impulses occur when $2n-4m=0\Rightarrow n=2m$.

Since m only takes on integer values and $n=2m$ the impulses occur when n is an integer multiple of 2.

$$18\delta_5[3n]=18\sum_{m=-\infty}^{\infty}\delta[3n-5m]$$, Impulses occur when $3n-5m=0\Rightarrow n=5m/3$.

Since n and m must be integers, the impulses occur when $5m/3$ is an integer or when m is an integer multiple of 3 making n an integer multiple of 5. So the fundamental period of $\mathrm{x}[n]$ is the LCM of 2 and 5 which is 10.

$$P_x=\frac{1}{10}\sum_{n=0}^{9}\left|\mathrm{x}[n]\right|^2=\frac{1}{10}\left[(-7)^2+0^2+11^2+0^2+11^2+(-18)^2+11^2+0^2+11^2+0^2\right]$$

$$P_x=\frac{49+121\times4+324}{10}=85.7$$

Solution of EECS 315 Test 5 F13

1. The impulse response of a digital filter is $\mathrm{x}[n] = 3.2(0.4)^n \mathrm{u}[n]$. Find the numerical signal energy of $\mathrm{x}[n]$, E_x.

$$E_x = \sum_{n=-\infty}^{\infty} |\mathrm{x}[n]|^2 = 3.2^2 \sum_{n=0}^{\infty} (0.4^n)^2 = 10.24 \sum_{n=0}^{\infty} (0.4^2)^n$$

$$E_x = 10.24 \sum_{n=0}^{\infty} 0.16^n = 10.24 \frac{1}{1-0.16} = 12.19$$

2. Find the numerical average signal power of $\mathrm{x}[n] = 7\delta_4[2n] - 15\delta_5[3n]$.

Using the definition of the discrete-time periodic impulse, $\delta_N[n] = \sum_{m=-\infty}^{\infty} \delta[n - mN]$,

$$7\delta_4[2n] = 7 \sum_{m=-\infty}^{\infty} \delta[2n - 4m]$$, Impulses occur when $2n - 4m = 0 \Rightarrow n = 2m$.

Since m only takes on integer values and $n = 2m$ the impulses occur when n is an integer multiple of 2.

$$15\delta_5[3n] = 15 \sum_{m=-\infty}^{\infty} \delta[3n - 5m]$$, Impulses occur when $3n - 5m = 0 \Rightarrow n = 5m/3$.

Since n and m must be integers, the impulses occur when $5m/3$ is an integer or when m is an integer multiple of 3 making n an integer multiple of 5. So the fundamental period of $\mathrm{x}[n]$ is the LCM of 2 and 5 which is 10.

$$P_x = \frac{1}{10} \sum_{n=0}^{9} |\mathrm{x}[n]|^2 = \frac{1}{10}\left[(-8)^2 + 0^2 + 7^2 + 0^2 + 7^2 + (-15)^2 + 7^2 + 0^2 + 7^2 + 0^2\right]$$

$$P_x = \frac{64 + 49 \times 4 + 225}{10} = 48.5$$