Solution to Test #3 in ECE 315 F02

1. A DT function is defined by $\mathrm{g}[n] = \left(a + (-1)^n\right)\mathrm{u}[n]$. Another DT function, $\mathrm{h}[n]$ is defined as the accumulation of $\mathrm{g}[n]$ from $-\infty$ to n. Find the numerical value of $\mathrm{h}[n_0]$.

$$\mathrm{h}[n] = \sum_{m=-\infty}^{n} \left(a + (-1)^m\right)\mathrm{u}[m] = \sum_{m=0}^{n}\left(a + (-1)^m\right) = \sum_{m=0}^{n} a + \sum_{m=0}^{n} (-1)^m = a + 1 + \begin{Bmatrix} 1 \ , \ n \text{ odd} \\ 0 \ , \ n \text{ even} \end{Bmatrix}$$

Therefore

$$\mathrm{h}[n_0] = a + 1 + \begin{Bmatrix} 1 \ , \ n_0 \text{ odd} \\ 0 \ , \ n_0 \text{ even} \end{Bmatrix} .$$

2. A CT signal is described by $\mathrm{x}(t) = A\mathrm{rect}(t) + B\mathrm{rect}(t - 0.5)$. What is its signal energy?

$$E_x = \int_{-\infty}^{\infty} \left|A\mathrm{rect}(t) + B\mathrm{rect}(t - 0.5)\right|^2 dt$$

Since these are purely real functions,

$$E_x = \int_{-\infty}^{\infty} \left(A\mathrm{rect}(t) + B\mathrm{rect}(t - 0.5)\right)^2 dt$$

$$E_x = \int_{-\infty}^{\infty} \left(A^2\mathrm{rect}^2(t) + B^2\mathrm{rect}^2(t - 0.5) + 2AB\mathrm{rect}(t)\mathrm{rect}(t - 0.5)\right) dt$$

$$E_x = A^2 \int_{-\frac{1}{2}}^{\frac{1}{2}} dt + B^2 \int_{0}^{1} dt + 2AB \int_{0}^{\frac{1}{2}} dt = A^2 + B^2 + AB$$

3. Find the average signal power of a periodic DT signal. It is simply the sum of the squares of all the impulse strengths in exactlyone period, divided by the period.