Test Solution Summary - S02

Below are plotted a CT function, $g_1(t)$, and a DT function, $g_1[n]$. Both functions are zero for all time outside the range plotted below. Let some other functions be defined by

$$g_2(t) = 3g_1(2-t) \quad , \quad g_3(t) = -2g_1\left(\frac{t}{4}\right) \quad , \quad g_4(t) = g_1\left(\frac{t-3}{2}\right)$$

$$g_2[n] = -g_1[2n] \quad , \quad g_3[n] = 2g_1[n-2] \quad , \quad g_4[n] = 3g_1\left[\frac{n}{3}\right]$$

Fill in the blanks. (If a value is undefined just write "undefined" in the blank.)

(a) $g_2(1) =$ -3

(b) $g_3(-1) =$ -3.5

(c) $g_3[0] =$ 6

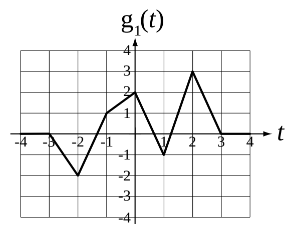
(d) $g_4[2] =$ Undefined

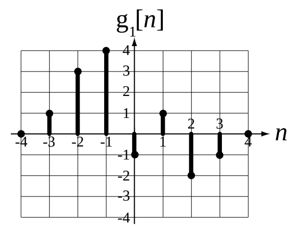
(e) $[g_4(t)g_3(t)]_{t=2} = \frac{3}{2} \times (-1) = -\frac{3}{2}$

(f) $\left(\frac{g_2[n]}{g_3[n]}\right)_{n=-1} = \frac{-3}{2} = -\frac{3}{2}$

(g) $\int_{-3}^{-1} g_4(t)\,dt =$ -2 The function is linear between those limits and the area under it is a triangle. The base width is 2 and the height is -2. Therefore the area is -2.

(h) $\sum_{n=-1}^{1} g_2[n] = g_2[-1] + g_2[0] + g_2[1] = -3 + 1 + 2 = 0$

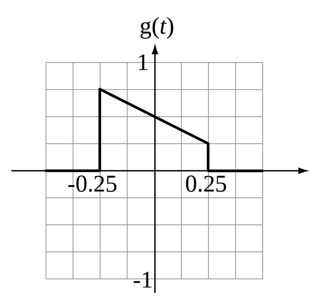




Sketch, in the space provided, the function

$$g(t) = \text{rect}(2t)\,\text{tri}\left(t + \frac{1}{2}\right) \quad .$$

Put numerical vertical and horizontal scales on the graph so that actual numerical values of the function could be read from the sketch.



Find the signal energy of

$$x[n] = \text{rect}_{10}[n]\,\text{comb}_3[n] \quad .$$

$$E_x = \sum_{n=-\infty}^{\infty} |x[n]|^2 = \sum_{n=-\infty}^{\infty} |\text{rect}_{10}[n]\,\text{comb}_3[n]|^2 = \sum_{n=-10}^{10} |\text{comb}_3[n]|^2$$

$$E_x = \sum_{n=-\infty}^{\infty} \left(\delta[n+9]+\delta[n+6]+\delta[n+3]+\delta[n]+\delta[n-3]+\delta[n-6]+\delta[n-9]\right)^2$$

$$E_x = 7$$

If $\mathrm{g}(t) = 4\sin\left(\frac{\pi t}{8}\right) * \delta(t-4)$,

$$\mathrm{g}(t) = 4\sin\left(\frac{\pi t}{8}\right) * \delta(t-4) = 4\sin\left(\frac{\pi(t-4)}{8}\right)$$

$$\mathrm{g}(-1) = 4\sin\left(\frac{\pi(-1-4)}{8}\right) = 4\sin\left(-\frac{5\pi}{8}\right) = -3.696$$

If $\mathrm{g}[n] = 10\cos\left(\frac{2\pi n}{12}\right) * \delta[n+8]$,

$$\mathrm{g}[n] = 10\cos\left(\frac{2\pi(n+8)}{12}\right)$$

$$\mathrm{g}[4] = 10\cos\left(\frac{2\pi(4+8)}{12}\right) = 10\cos(2\pi) = 10$$

If $\mathrm{g}(t) = -5\,\mathrm{rect}\left(\frac{t+4}{2}\right) * \delta(3t)$,

$$\mathrm{g}(t) = -\frac{5}{3}\mathrm{rect}\left(\frac{t+4}{2}\right) * \delta(t) = -\frac{5}{3}\mathrm{rect}\left(\frac{t+4}{2}\right)$$

$$\mathrm{g}(1) = -\frac{5}{3}\mathrm{rect}\left(\frac{1+4}{2}\right) = -\frac{5}{3}\mathrm{rect}\left(\frac{5}{2}\right) = 0$$

$$\mathrm{g}(-4) = -\frac{5}{3}\mathrm{rect}\left(\frac{-4+4}{2}\right) = -\frac{5}{3}$$

$$\mathrm{g}(1) - \mathrm{g}(-4) = 0 - \left(-\frac{5}{3}\right) = \frac{5}{3}$$

If $\mathrm{g}[n] = \mathrm{rect}_2[2n] * \left(\delta[n-1] - 2\delta[n-2]\right)$,

$$\mathrm{g}[n] = \left(\mathrm{rect}_2[2n] * \delta[n-1] - \mathrm{rect}_2[2n] * 2\delta[n-2]\right)$$

$$\mathrm{g}[n] = \left(\mathrm{rect}_2[2(n-1)] - 2\,\mathrm{rect}_2[2(n-2)]\right)$$

$$\mathrm{g}[2] = \left(\mathrm{rect}_2[2(2-1)] - 2\,\mathrm{rect}_2[2(2-2)]\right) = \left(\mathrm{rect}_2[2] - 2\,\mathrm{rect}_2[0]\right)$$

$$\mathrm{g}[2] = 1 - 2 = -1$$

If a function, $\mathrm{g}(t)$, with period, $T_0 = 6$, is represented exactly for all time by a CTFS,

$$\mathrm{G}[k] = 4\,\mathrm{sinc}^2\left(\frac{k}{3}\right) ,$$

and $T_F = T_0$,what is the CTFS of $-3\mathrm{g}(t+2)$?

$$\mathrm{g}(t) \xleftrightarrow{\mathcal{FS}} \mathrm{G}[k] = 4\,\mathrm{sinc}^2\left(\frac{k}{3}\right)$$

$$-3\mathrm{g}(t) \xleftrightarrow{\mathcal{FS}} -12\,\mathrm{sinc}^2\left(\frac{k}{3}\right)$$

$$-3\mathrm{g}(t+2) \xleftrightarrow{\mathcal{FS}} -12\,\mathrm{sinc}^2\left(\frac{k}{3}\right)e^{-j\frac{\pi k}{3}(-2)} = -12\,\mathrm{sinc}^2\left(\frac{k}{3}\right)e^{j\frac{2\pi k}{3}}$$

Identify which of these functions has a complex CTFS, $\mathrm{G}[k]$, for which 1) $\mathrm{Re}(\mathrm{G}[k]) = 0$ for all *k*, or 2) $\mathrm{Im}(\mathrm{G}[k]) = 0$ for all *k*, or 3) neither of these conditions applies.

(a) $\mathrm{g}(t) = 18\cos(200\pi t) + 22\cos(240\pi t)$ $\mathrm{Im}(\mathrm{G}[k]) = 0$ (Even function)

(b) $\mathrm{g}(t) = -4\sin(10\pi t)\sin(2000\pi t)$ $\mathrm{Im}(\mathrm{G}[k]) = 0$ (Even function)

(c) $\mathrm{g}(t) = \mathrm{tri}\left(\frac{t-1}{4}\right) * \frac{1}{10}\mathrm{comb}\left(\frac{t}{10}\right)$ Neither (Neither even nor odd function)

A periodic DT signal, $\mathrm{x}[n]$, is exactly described for all discrete time by its DTFS,

$$\mathrm{X}[k] = \left(\mathrm{comb}_8[k-1] + \mathrm{comb}_8[k+1] + j2\,\mathrm{comb}_8[k+2] - j2\,\mathrm{comb}_8[k-2]\right)e^{-j\frac{\pi k}{4}} .$$

(a) Write a correct analytical expression for $\mathrm{x}[n]$ in which $\sqrt{-1}$, "*j*", does not appear.

$$\cos\left(\frac{2\pi n}{8}\right) \xleftrightarrow{\mathcal{FS}} \frac{1}{2}\left(\mathrm{comb}_8[k-1] + \mathrm{comb}_8[k+1]\right)$$

$$2\cos\left(\frac{2\pi n}{8}\right) \xleftrightarrow{\mathcal{FS}} \mathrm{comb}_8[k-1] + \mathrm{comb}_8[k+1]$$

$$\sin\left(\frac{4\pi n}{8}\right) = \sin\left(\frac{2\pi n}{4}\right) \xleftrightarrow{FS} \frac{j}{2}\left(\mathrm{comb}_8[k+2] - \mathrm{comb}_8[k-2]\right)$$

$$4\sin\left(\frac{2\pi n}{4}\right) \xleftrightarrow{FS} j2\left(\mathrm{comb}_8[k+2] - \mathrm{comb}_8[k-2]\right)$$

$$2\cos\left(\frac{2\pi(n-1)}{8}\right) + 4\sin\left(\frac{2\pi(n-1)}{4}\right) \xleftrightarrow{\mathcal{FS}} \begin{pmatrix}\mathrm{comb}_8[k-1] + \mathrm{comb}_8[k+1] \\ +j2\,\mathrm{comb}_8[k+2] - j2\,\mathrm{comb}_8[k-2]\end{pmatrix}e^{-j2\pi\frac{k}{8}}$$

Therefore

$$\mathrm{x}[n] = 2\cos\left(\frac{2\pi(n-1)}{8}\right) + 4\sin\left(\frac{2\pi(n-1)}{4}\right)$$

(b) What is the numerical value of $\mathrm{x}[n]$ at $n = -10$?

$$\mathrm{x}[-10] = 2\cos\left(\frac{2\pi(-10-1)}{8}\right) + 4\sin\left(\frac{2\pi(-10-1)}{4}\right) = 2\cos\left(-\frac{11\pi}{4}\right) + 4\sin\left(-\frac{11\pi}{2}\right)$$

$$\mathrm{x}[-10] = 2\cos\left(\frac{11\pi}{4}\right) - 4\sin\left(\frac{11\pi}{2}\right) = 2\cos\left(\frac{3\pi}{4}\right) - 4\sin\left(\frac{3\pi}{2}\right) = 2\left(-\frac{1}{\sqrt{2}}\right) - 4(-1)$$

$$\mathrm{x}[-10] = 4 - \frac{2}{\sqrt{2}} = \frac{4\sqrt{2}-2}{\sqrt{2}} = \frac{3.657}{1.414} = 2.586$$

For each time-domain function, find its CTFT and give the numerical value of the CTFT at the specified frequency.

(a) $\mathrm{x}(t) = 3\mathrm{rect}(t-1)$

$$\mathrm{X}(f)=3\,\mathrm{sinc}(f)e^{-j2\pi f}\Rightarrow \mathrm{X}\left(\frac{1}{4}\right)=3\,\mathrm{sinc}\left(\frac{1}{4}\right)e^{-j2\pi\frac{1}{4}}=-j3\frac{\sin\left(\frac{\pi}{4}\right)}{\frac{\pi}{4}}=-j\frac{6\sqrt{2}}{\pi}=-j2.70$$

(b) $\mathrm{x}(t)=4\,\mathrm{sinc}^2(3t)$

$$\mathrm{X}(f)=\frac{4}{3}\mathrm{tri}\left(\frac{f}{3}\right)\Rightarrow \mathrm{X}(j\omega)=\frac{4}{3}\mathrm{tri}\left(\frac{\omega}{6\pi}\right)\Rightarrow \mathrm{X}(j4\pi)=\frac{4}{3}\mathrm{tri}\left(\frac{2}{3}\right)=\frac{4}{9}=0.444\ldots$$

(c) $\mathrm{x}(t)=\mathrm{rect}(t)*\mathrm{rect}(2t)$

$$\mathrm{X}(f)=\mathrm{sinc}(f)\frac{1}{2}\mathrm{sinc}\left(\frac{f}{2}\right)\Rightarrow \mathrm{X}\left(\frac{1}{2}\right)=\frac{1}{2}\mathrm{sinc}\left(\frac{1}{2}\right)\mathrm{sinc}\left(\frac{1}{4}\right)=\frac{1}{2}\frac{\sin\left(\frac{\pi}{2}\right)}{\frac{\pi}{2}}\frac{\sin\left(\frac{\pi}{4}\right)}{\frac{\pi}{4}}=\frac{1}{2}\frac{\frac{\sqrt{2}}{2}}{\frac{\pi^2}{8}}=2\frac{\sqrt{2}}{\pi^2}=0.287$$

For each frequency-domain function, find its inverse CTFT and give the numerical value of the inverse CTFT at the specified time.

(a) $\mathrm{X}(f)=10\left[\delta\left(f-\frac{1}{2}\right)+\delta\left(f+\frac{1}{2}\right)\right]$

$$\mathrm{x}(t)=20\cos(\pi t)\Rightarrow \mathrm{x}(1)=20\cos(\pi)=-20$$

(b) $\mathrm{X}(j\omega)=-2\,\mathrm{sinc}\left(\frac{\omega}{2\pi}\right)*3\,\mathrm{sinc}\left(\frac{\omega}{\pi}\right)$

$$\mathrm{X}(f)=-2\,\mathrm{sinc}(f)*3\,\mathrm{sinc}(2f)\Rightarrow \mathrm{x}(t)=-6\,\mathrm{rect}(t)\frac{1}{2}\mathrm{rect}\left(\frac{t}{2}\right)\Rightarrow \mathrm{x}(0)=-3\,\mathrm{rect}(0)\,\mathrm{rect}\left(\frac{0}{2}\right)=-3$$

Find an expression for the DTFT, $\mathrm{X}(F)$ or $\mathrm{X}(j\Omega)$, of these functions and evaluate it at the DT frequency, F or Ω, indicated.

(a) $\mathrm{x}[n]=4\left(\frac{2}{3}\right)^n\mathrm{u}[n]$

$$\mathrm{X}(j\Omega)=\frac{4}{1-\frac{2}{3}e^{-j\Omega}}$$

$$\mathrm{X}(j\pi)=\frac{4}{1-\frac{2}{3}e^{-j\pi}}=\frac{4}{1+\frac{2}{3}}=\frac{12}{5}=2.4$$

(b) $\mathrm{x}[n]=2\,\mathrm{rect}_3[n-2]$

$$\mathrm{X}(F)=2\frac{\sin(7\pi F)}{\sin(\pi F)}e^{-j4\pi F}$$

$$\mathrm{X}\left(\frac{1}{8}\right)=2\frac{\sin\left(\frac{7\pi}{8}\right)}{\sin\left(\frac{\pi}{8}\right)}e^{-j\frac{\pi}{2}}=-j2$$

Find an expression for the inverse DTFT, $\mathrm{x}[n]$, of this function and evaluate it at the discrete time, n, indicated.

$$\mathrm{X}(F)=[\mathrm{rect}(10F)*\mathrm{comb}(F)]\otimes\frac{1}{2}\left[\mathrm{comb}\left(F-\frac{1}{4}\right)+\mathrm{comb}\left(F+\frac{1}{4}\right)\right]$$

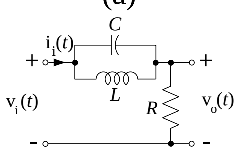
$$\mathrm{x}[n]=\frac{1}{10}\operatorname{sinc}\left(\frac{n}{10}\right)\cos\left(\frac{2\pi n}{4}\right)$$

$$\mathrm{x}[2]=\frac{1}{10}\operatorname{sinc}\left(\frac{2}{10}\right)\cos(\pi)=\frac{1}{10}\frac{\sin\left(\frac{\pi}{5}\right)}{\frac{\pi}{5}}\cos(\pi)=-0.09355$$

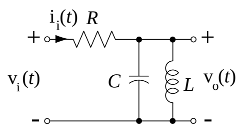
For each circuit below the transfer function is the ratio, $\mathrm{H}(f)=\dfrac{\mathrm{V}_o(f)}{\mathrm{V}_i(f)}$. Write in each blank the letter designations of all circuits that fit the description to the left. (If no circuits fit just write "none".) (2 pts each)

1. Zero transfer function at $f=0$ — b,c,e,h
2. Zero transfer function at $f\to+\infty$ — b,d,f,h
3. Transfer function magnitude of one at $f=0$ — a,d,f,g
4. Transfer function magnitude of one at $f\to+\infty$ — a,c,e,g
5. Transfer function magnitude non-zero and phase of zero at some frequency, $0<f<\infty$, (at a finite, non-zero frequency). — b,h

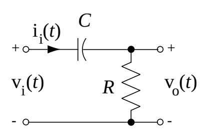
(a)



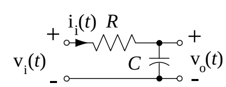
(b)



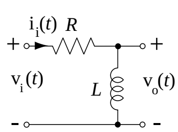
(c)



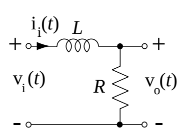
(d)



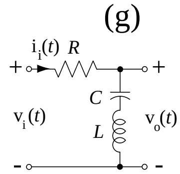
(e)



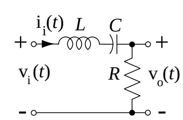
(f)



(g)



(h)



Find the Nyquist rates for these signals. (If the Nyqist rate is infinite just write "infinity".)

(a) $\mathrm{x}(t)=-200\cos(50\pi t)$ — 50 Hz

$$\mathrm{X}(f)=-100[\delta(f-25)+\delta(f+25)]$$

(b) $\mathrm{x}(t)=40\cos(20\pi t)-30\sin(100\pi t)$ — 100 Hz

$$\mathrm{X}(f)=20[\delta(f-10)+\delta(f+10)]-j15[\delta(f+50)-\delta(f+50)]$$

(c) $\mathrm{x}(t)=25\cos(20\pi t)\sin(100\pi t)$ — 120 Hz

$$\mathrm{X}(f)=j\frac{25}{4}[\delta(f-10)+\delta(f+10)]*[\delta(f+50)-\delta(f-50)]$$

$$\mathrm{X}(f)=j\frac{25}{4}[\delta(f+40)+\delta(f+60)-\delta(f-60)-\delta(f-40)]$$

(d) $\mathrm{x}(t)=\operatorname{sinc}(t)*\operatorname{comb}\left(\frac{t}{5}\right)$ — 0.8 Hz

$$\mathrm{X}(f)=\operatorname{rect}(f)5\operatorname{comb}(5f)=\sum_{k=-\infty}^{\infty}\operatorname{rect}\left(\frac{k}{5}\right)\delta\left(f-\frac{k}{5}\right)=\sum_{k=-2}^{2}\delta\left(f-\frac{k}{5}\right)$$

(e) $\mathrm{x}(t) = \mathrm{tri}(t) * \mathrm{comb}\left(\frac{t}{5}\right)$ Infinity

$\mathrm{X}(f) = 5\,\mathrm{sinc}^2(f)\,\mathrm{comb}(5f)$

A periodic continuous-time signal, $\mathrm{x}(t)$, is sampled at $f_s = 20\text{ Hz}$, which is exactly 4 times its fundamental frequency, ($f_s = 4f_0$), to form a discrete-time signal, $\mathrm{x}[n]$. Let the first four values of $\mathrm{x}[n]$ starting at discrete time, $n = 0$, be

$$\mathrm{x}[0] = 2 \quad \mathrm{x}[1] = -1 \quad \mathrm{x}[2] = -2 \quad \mathrm{x}[3] = a.$$

A discrete Fourier transform (DFT), $\mathrm{X}[k]$, of these samples is found.

(a) If $\mathrm{X}[0] = 6$, what is the numerical value of a?

$$\mathrm{X}[0] = \sum_{n=0}^{3} \mathrm{x}[n] = 2 - 1 - 2 + a = 6 \Rightarrow a = 7$$

(b) If $\mathrm{X}[1] = b + j2$ what are the numerical values of a, b, $\mathrm{X}[0]$ and $\mathrm{X}[3]$?

$$\mathrm{X}[1] = \sum_{n=0}^{3} \mathrm{x}[n] e^{-j\pi\frac{n}{2}} = 2 + j + 2 + ja = b + j2 \Rightarrow b = 4 \text{ and } a = 1$$

Then

$$\mathrm{X}[0] = \sum_{n=0}^{3} \mathrm{x}[n] = 2 - 1 - 2 + 1 = 0$$

and $\mathrm{X}[3] = \mathrm{X}^*[1] = 4 - j2$.

(c) What is the numerical value of $\mathrm{x}(t)$ at time, $t = 0.65$?

The period of $\mathrm{x}(t)$ is 0.2 seconds and the time between samples is $T_s = \frac{1}{f_s} = 0.05$.

Since $\mathrm{x}(t)$ is periodic

$\mathrm{x}(0.65) = \mathrm{x}(0.65 - 3 \times 0.2) = \mathrm{x}(0.05) = \mathrm{x}(T_s) = \mathrm{x}[1] = -1$

In tossing one fair (not loaded) die there are 6 distinct outcomes, the numbers 1, 2, 3, 4, 5 and 6. The probability of each outcome is $\frac{1}{6}$

1. Four fair dice are tossed simultaneously. What is the probability that all four will show a “2”?

$$\Pr(\text{all four show a "2"}) = \left(\frac{1}{6}\right)^4 = \frac{1}{1296} = 0.0007716$$

2. Four fair dice are tossed simultaneously. What is the probability that die #1 will show a “1”, die #2 will show a “5”, die #3 will show a “3” and die #4 will show a “6”?

$$\Pr(\{1,5,3,6\}) = \left(\frac{1}{6}\right)^4 = \frac{1}{1296} = 0.0007716$$

3. In an experiment three fair dice are tossed simultaneously. The outcome is defined as the set,

$$\{\text{die \#1outcome,die \#2outcome,die \#3outcome}\}\ .$$

How many distinct outcomes are there? $6^3 = 216$

4. Two fair dice are tossed simultaneously. What is the probability that both dice show a number less than three?

$$\Pr(\text{number } < \text{ 3 for one die}) = \frac{2}{6} = \frac{1}{3}$$

$$\Pr(\text{number } < \text{ 3 for two dice simultaneously}) = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$$

5. Two fair dice are tossed simultaneously. What is the probability that both dice show the same number?

$$\Pr(\text{both dice showing the same number}) = \begin{bmatrix} \Pr(\text{both showing a "1"}) \\ +\Pr(\text{both showing a "2"}) \\ +\cdots \\ +\Pr(\text{both showing a "6"}) \end{bmatrix}$$

$$\Pr(\text{both dice showing the same number}) = \frac{1}{36} + \frac{1}{36} + \cdots + \frac{1}{36} = \frac{6}{36} = \frac{1}{6}$$

Answer the following questions about the pdf's below:

Which one is for the random variable which has an expected value with the smallest magnitude ? (d)

Which one is for the random variable which has the smallest standard deviation?(c)

Which one(s) are for a mixed random variable (as opposed to a discrete-value or continuous-value random variable)? (e)

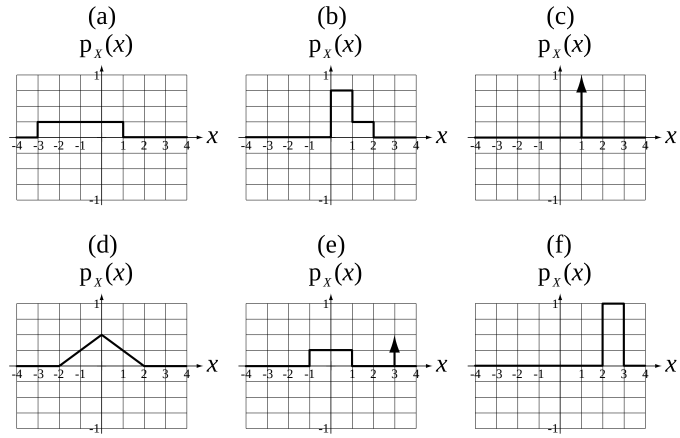
Usually, for a random variable, $\mathrm{E}(X^2) \neq [\mathrm{E}(X)]^2$. Is it true for any of the random variables whose pdf's are shown below that $\mathrm{E}(X^2) = [\mathrm{E}(X)]^2$? If so which one(s)? (c)

What is the expected value, $\mathrm{E}(X)$, of the random variable whose pdf is in (e) ?

$$\mathrm{E}(X) = \int_{-\infty}^{\infty} \left[\frac{x}{4}\mathrm{rect}\left(\frac{x}{2}\right) + \frac{x}{2}\delta(x-3)\right]dx = \frac{1}{4}\int_{-1}^{1} x\,\mathrm{rect}\left(\frac{x}{2}\right)dx + \int_{-\infty}^{\infty}\frac{x}{2}\delta(x-3)dx = 0 + \frac{1}{2}(3) = \frac{3}{2}$$

If the random variables whose pdf's are in (c) and (f) are added, what is the expected value of the sum? The sum of the expected values, $\frac{7}{2}$.

Given that the variance of a uniform distribution of width, w, is $\frac{w^2}{12}$, if the random variables whose pdf's are in (c) and (a) are added what is the variance of their sum? The pdf's are convolved to form the pdf of the sum. Being convolved with a unit impulse does not change the width so the variance is $\frac{16}{12} = \frac{4}{3}$.



"Bandlimited white noise" is the name for the response of an ideal lowpass filter excited by white noise. Let the white noise excitation have an autocorrelation function,

$$\mathrm{R}_x(\tau) = 10\delta(\tau)$$

and let the ideal lowpass filter's transfer function be

$$\mathrm{H}(f) = 4\,\mathrm{rect}\left(\frac{f}{40}\right) .$$

(a) Find the average signal power of the response of the ideal lowpass filter.

$$\mathrm{G}_x(f) = 10$$

$$\mathrm{G}_y(f) = \mathrm{G}_x(f)\left|\mathrm{H}(f)\right|^2 = 10\left|4\,\mathrm{rect}\left(\frac{f}{40}\right)\right|^2 = 10 \times 16\,\mathrm{rect}^2\left(\frac{f}{40}\right) = 160\,\mathrm{rect}\left(\frac{f}{40}\right)$$

$$P_y = \int_{-\infty}^{\infty} \mathrm{G}_y(f)\,df = \int_{-\infty}^{\infty} 160\,\mathrm{rect}\left(\frac{f}{40}\right)df = 160\int_{-20}^{20} df = 160 \times 40 = 6400$$

(b) Find the autocorrelation function of the bandlimited white noise response of the filter.

$$\mathrm{R}_y(\tau) \xleftrightarrow{\mathcal{F}} \mathrm{G}_y(f)$$

$$6400\,\mathrm{sinc}(40\tau) \xleftrightarrow{\mathcal{F}} 160\,\mathrm{rect}\left(\frac{f}{40}\right)$$

$$\mathrm{R}_y(\tau) = 6400\,\mathrm{sinc}(40\tau)$$