Solution to ECE Test #1 Su05 #1

1. A continuous-time signal is always continuous. False

2. The fundamental period of a periodic signal is the reciprocal of <u>the fundamental cyclic frequency</u>.

3. The generalized derivative of the unit step is <u>the unit impulse</u>.

4. The derivative of <u>unit ramp</u> is the unit step.

5. Name a function of continuous time t for which the two successive transformations $t \to -t$ and $t \to t-1$ leave the function unchanged. $\cos(2\pi t)$, $\text{comb}(t)$, etc... (Any even periodic function with a fundamental period of one.)

6. Find the numerical values of these functions.

(a) $3\,\text{sinc}(3.5) = 3\dfrac{\sin(3.5\pi)}{3.5\pi} = -0.27284$

(b) $\displaystyle\int_{-\infty}^{1/2} \text{rect}(t/2)\,dt = \int_{-1}^{1/2} dt = 3/2 = 1.5$

(c) $\text{tri}(3/4) - \text{tri}(1/2) = (1-|3/4|) - (1-|1/2|) = -1/4$

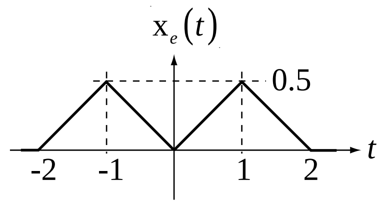
(d) $\displaystyle\int_{0}^{10} \delta(t-3)\cos(\pi t/10)\,dt = \cos(3\pi/10) = 0.5878$

(e) $\displaystyle\int_{-5}^{5} \delta(t+8)\cos(\pi t/10)\,dt = \underline{0}$ The impulse is not within the range of integration.

(f) $\displaystyle\int_{-5}^{5} \sin(\pi t/20)\,dt = \underline{0}$ Odd function integrated over symmetrical limits.

7. Find and sketch the even part of $\text{x}(t) = \text{tri}(t-1)$. The sketch must have a scale so that approximate numerical values could be read from it.

The even part is $\text{x}_e(t) = \dfrac{\text{tri}(t-1) + \text{tri}(-t-1)}{2} = \dfrac{\text{tri}(t-1) + \text{tri}(t+1)}{2}$.



8. Find the fundamental periods of these signals. If a signal is not periodic just write "infinity".

(a) $x(t) = -5\sin(20\pi t) + 2\cos(15\pi t)$ The individual fundamental frequencies are 10 and 7.5. The GCD of those is 2.5 which is the fundamental frequency of $x(t)$. Therefore the fundamental period of $x(t)$ is $T_0 = 1/2.5 = 0.4$

(b) $x(t) = 3\sin(20t) + 8\cos(4t)$ The individual fundamental frequencies are $10/\pi$ and $2/\pi$. The GCD of those is $2/\pi$ which is the fundamental frequency of $x(t)$. Therefore the fundamental period of $x(t)$ is $T_0 = 1/(2/\pi) = \pi/2 \cong 1.57$

(c) $x(t) = 10\sin(20t) + 7\cos(10\pi t)$ The ratio of the two fundamental frequencies is not a rational number so $x(t)$ is not periodic. $T_0 = \text{infinity}$

9. The first backward difference of the unit sequence is the DT unit impulse.

10. What is the numerical maximum (most positive) value of $u[n] + u[-n]$? 2

All the values of this function are one except when $n = 0$. At $n = 0$ the value is two.

11. Find the numerical values of these functions.

(a) $\text{ramp}[6] - u[-2] = 6 - 0 = 6$

(b) $\sum_{n=-\infty}^{7} \text{rect}_9[n]$ This is the sum of unit impulses starting at $n = -9$ and continuing through $n = 7$. That is 17 unit impulses. So the answer is 17.

(c) $g[4]$ where $g[n] = \sin(2\pi(n-3)/8) + \delta[n-3]$

$$g[4] = \sin(2\pi(4-3)/8) + \delta[4-3] = \sin(\pi/4) + \delta[1] = \pi/4 \cong 0.707$$

12. The even part of $g[n]$ is $g_e[n] = \text{tri}(n/16)$ and the odd part of $g[n]$ is $g_o[n] = u[n+3] - u[n] - u[n-1] + u[n-4]$. Find the numerical value of $g[3]$.

$$\mathrm{g}[3] = \mathrm{tri}(3/16) + \mathrm{u}[3+3] - \mathrm{u}[3] - \mathrm{u}[3-1] + \mathrm{u}[3-4]$$

$$\mathrm{g}[3] = 13/16 + 1 - 1 - 1 + 0 = -3/16 = -0.1875$$

13. Find the fundamental periods of these signals. If a signal is not periodic just write "infinity".

(a) $\mathrm{x}[n] = 3\sin(2\pi n/12) + \cos(2\pi n/20)$ The two fundamental periods are 12 and 20. The LCM of 12 and 20 is 60. Therefore $N_0 = 60$.

(b) $\mathrm{x}[n] = 3\sin(n/16) + 8\cos(n/6)$ The factor K in the form $\sin(2\pi K n)$ is not rational. (It is $1/32\pi$ for one and $1/12\pi$ for the other. Therefore both functions are aperiodic and their sum is also. $N_0 =$ infinity

14. Identify each signal as either an energy signal or a power signal and find the signal energy of all energy signals and the average signal power of all power signals.

(a) $\mathrm{x}[n] = \mathrm{u}[n] - 2\,\mathrm{u}[n-4] + \mathrm{u}[n-10]$ Energy Signal

The signal energy is the sum of the squares of all the impulses in the signal. The signal has the values 1 for $0 \le n < 4$, -1 for $4 \le n < 10$ and zero elsewhere. So the signal energy is the sum of the squares of those values which is 10.

(b) $\mathrm{x}[n] = -4\cos(\pi n)$ Power Signal

Since n is an integer, the signal $\mathrm{x}[n] = -4\cos(\pi n)$ can be written as $\mathrm{x}[n] = -4(-1)^n$ and the square of its magnitude can be written as $|\mathrm{x}[n]|^2 = \left(-4(-1)^n\right)^2 = 16(-1)^{2n} = 16$. The signal power is the average of this squared magnitude which is 16.

(c) $\mathrm{x}(t) = 2\sin(\pi t/2)\,\mathrm{rect}(t/8)$ Energy Signal

Signal Energy = $\displaystyle\int_{-4}^{4} |2\sin(\pi t/2)|^2\, dt = 8\int_0^4 \sin^2(\pi t/2)\,dt = 8\int_0^4 \left[\frac{1}{2} - \frac{\cos(\pi t)}{2}\right] dt$

$$= 8\left[2 - \frac{\sin(\pi t)}{2\pi}\right]_0^4 = 8[2-0] = 16$$

(d) $\mathrm{x}(t) = 2\,\mathrm{sgn}(3\cos(2\pi t))$ Power Signal

$\mathrm{x}(t) = 2\,\mathrm{sgn}(3\cos(2\pi t))$ is always either +2 or -2 and its square is a constant 4. Therefore signal Power = 4