Solution of ECE 315 Test 3 F05

Let two DT signals be defined by

$$g_1[n] = \begin{cases} 0 \ , \ n < -3 \\ 2n \ , \ -3 \le n < 3 \\ 0 \ , \ n \ge 3 \end{cases} \quad \text{and} \quad g_2[n] = -3\sin(2\pi n/4)$$

(a) Find the numerical signal energy E_1 of $g_1[n]$.

E_1 = <u>76</u>

$$E_1 = \sum_{n=-\infty}^{\infty} |g_1[n]|^2 = \sum_{n=-3}^{2} |2n|^2 = 4\left[(-3)^2 + (-2)^2 + (-1)^2 + 0^2 + 1^2 + 2^2\right]$$

$$E_1 = 4 \times 19 = 76$$

(b) Find the numerical signal power P_2 of $g_2[n]$.

P_2 = <u>4.5</u>

$$P_2 = \frac{1}{N_0} \sum_{n=\langle N_0 \rangle} |g_2[n]|^2 = \frac{1}{4}\sum_{n=0}^{3} |-3\sin(2\pi n/4)|^2 = 2 \times \frac{9}{4}\sum_{n=0}^{1} \sin^2(2\pi n/4)$$

$$P_2 = \frac{9}{2}\left[\underbrace{\sin^2(0)}_{=0} + \underbrace{\sin^2(\pi/2)}_{1}\right] = \frac{9}{2} = 4.5$$

(c) If we perform the transformation $n \to 2n$ on $g_2[n]$ to form $g_3[n]$ and then perform the transformation $n \to n-1$ on $g_3[n]$ to form $g_4[n]$, what is the numerical signal power P_4 of $g_4[n]$? P_4 = <u>0</u>

$$g_3[n] = [-3\sin(2\pi n/4)]_{n \to 2n} = -3\sin(4\pi n/4) = -3\underbrace{\sin(\pi n)}_{=0\text{ , for any } n} = 0$$

$$g_4[n] = [0]_{n \to n-1} = 0$$

(d) If we reverse the order of the two transformations of part (c) what is the new numerical signal power P_{4new} of the new $g_4[n]$? P_{4new} = <u>9</u>

$$g_3[n] = [-3\sin(2\pi n/4)]_{n \to n-1} = -3\sin(2\pi(n-1)/4)$$

$$g_3[n] = -3\sin(2\pi n/4 - \pi/2) = 3\cos(2\pi n/4)$$

$$g_4[n] = [3\cos(2\pi n/4)]_{n \to 2n} = 3\underbrace{\cos(\pi n)}_{=1\text{ , for any } n} = 3$$

The signal power is the average of the square of the magnitude of the signal. In this case that is obviously 9.

Solution of ECE 315 Test 3 F05

Let two DT signals be defined by

$$g_1[n] = \begin{cases} 0 \ , \ n < -3 \\ 3n \ , \ -3 \le n < 3 \\ 0 \ , \ n \ge 3 \end{cases} \quad \text{and} \quad g_2[n] = -2\sin(2\pi n/4)$$

(a) Find the numerical signal energy E_1 of $g_1[n]$.

E_1 = 171

$$E_1 = \sum_{n=-\infty}^{\infty} |g_1[n]|^2 = \sum_{n=-3}^{2} |3n|^2 = 9\left[(-3)^2 + (-2)^2 + (-1)^2 + 0^2 + 1^2 + 2^2\right]$$

$$E_1 = 9 \times 19 = 171$$

(b) Find the numerical signal power P_2 of $g_2[n]$.

P_2 = 2

$$P_2 = \frac{1}{N_0} \sum_{n=\langle N_0 \rangle} |g_2[n]|^2 = \frac{1}{4}\sum_{n=0}^{3} |-2\sin(2\pi n/4)|^2 = 2 \times \frac{4}{4}\sum_{n=0}^{1} \sin^2(2\pi n/4)$$

$$P_2 = 2\left[\underbrace{\sin^2(0)}_{=0} + \underbrace{\sin^2(\pi/2)}_{1}\right] = 2$$

(c) If we perform the transformation $n \to 2n$ on $g_2[n]$ to form $g_3[n]$ and then perform the transformation $n \to n-1$ on $g_3[n]$ to form $g_4[n]$, what is the numerical signal power P_4 of $g_4[n]$? P_4 = 0

$$g_3[n] = \left[-2\sin(2\pi n/4)\right]_{n \to 2n} = -2\sin(4\pi n/4) = -2\sin(\pi n) = 0$$

$$g_4[n] = [0]_{n \to n-1} = 0$$

The signal power is the average of the square of the magnitude of the signal. In this case that is obviously 0.

(d) If we reverse the order of the two transformations of part (c) what is the new numerical signal power P_{4new} of the new $g_4[n]$? P_{4new} = 4

$$g_3[n] = \left[-2\sin(2\pi n/4)\right]_{n \to n-1} = -2\sin(2\pi(n-1)/4)$$

$$g_3[n] = -2\sin(2\pi n/4 - \pi/2) = 2\cos(2\pi n/4)$$

$$g_4[n] = \left[2\cos(2\pi n/4)\right]_{n \to 2n} = 2\cos(\pi n) = 2$$

The signal power is the average of the square of the magnitude of the signal. In this case that is obviously 4.