Solution of ECE 315 Test #1 Su03

I.   1 point each

1.   A CT function is time inverted and looks the same as it did before it was time inverted. What kind of function is it?

Even function
Answers like "rect" or "sinc" were counted wrong because the information does not specify a function that closely. The function *could be* a rectangle or sinc or comb, but it *must* be even.

2.   True   or   False ?
A continuous function is guaranteed to be a continuous-time function.

True
A continuous-time function need not be continuous, but a continuous function must be continuous-time.

3.   True   or   False ?
A fundamental period is a period but a period is not necessarily a fundamental period.

True
Any positive integer multiple of the fundamental period is a period of a function. The integer multiple, "1", yields the fundamental period.

4.   If $g(t) = 3u(1-t)$, what is the numerical value of $g(-2)$?

3
$g(-2) = 3u(1-(-2)) = 3\underbrace{u(3)}_{=1} = 3$

5.   What is the numerical value of $\int\limits_{-\infty}^{\infty} \delta(t+4)\,\mathrm{ramp}(t)\,dt$?

Zero
By the sampling property of the impulse,

$$\int\limits_{-\infty}^{\infty} \delta(t+4)\,\mathrm{ramp}(t)\,dt = \mathrm{ramp}(-4) = 0$$

6.   A CT function of time, t, is transformed by the transformation, $t \rightarrow 4t$. Compared with the original function, is the new (transformed) function compressed or expanded in time?

Compressed

7.   In the transformation, $g(t) \rightarrow Ag(bt - t_0)$, would it be better (quicker, easier) to do the time scaling before the time shifting or the time shifting before the time scaling?

Shifting first

$$g(t) \xrightarrow{g(t) \rightarrow A\,g(t)} Ag(t) \xrightarrow{t \rightarrow t - t_0} Ag(t - t_0) \xrightarrow{t \rightarrow bt} Ag(bt - t_0)$$

$$g(t) \xrightarrow{g(t) \rightarrow A\,g(t)} Ag(t) \xrightarrow{t \rightarrow bt} A\,g(bt) \xrightarrow{t \rightarrow t - t_0} Ag(b(t - t_0)) \neq A\,g(bt - t_0)$$

8.   A DT system is excited by a DT unit-ramp function and the response is unbounded. From these facts alone, it is impossible to determine whether the system is BIBO stable or not. Why?

Because the excitation (the unit-ramp) is unbounded and "BIBO" means "bounded-input-bounded-output". Since the excitation is unbounded, we cannot determine from these facts whether the system is BIBO stable.

9.   The excitation, x, of a DT system and the response, y, of that DT system are related by $y[n] = |x[n]|$. Is the system invertible?

No
If the response at some discrete time is some number, K, we don't know whether the excitation at that time was K or $-K$. Therefore, knowing the response does not allow us to uniquely determine the excitation.

II.

1.   The CT function, $g(t) = \text{rect}(t-3) + 2\text{rect}(t-7)$ is integrated in time from negative infinity to a definite time, t_0. On the assumption that there is no accumulated area in the integral before the time of occurrence of the first rectangle function, what is the numerical value of the integral if

(a)   (3 pts) $t_0 = 3$ The accumulated area is half the area of the first rectangle function or 1/2.

(b)   (3 pts) $t_0 = 10$ The accumulated area is the area of the first rectangle, 1, plus the area of the second rectangle which is 2. The total accumulated area is therefore 3.

2.   Which of the functions below are periodic and which are not?

(a)   (1 pt) $g(t) = \sin(20\pi t) + 4\cos(35\pi t)$ Periodic
Period is the LCM of 1/10 and 2/35 which is 0.4.

(b)   (1 pt) $g(t) = \sin(20\pi t) + 4\cos(35t)$ Aperiodic
Period would be the LCM of 1/10 and $2\pi/35$, but, because π is irrational, there is no finite LCM and the function is not periodic.

(c)   (1 pt) $g[n] = 2\text{tri}\left(\dfrac{n}{6}\right)$ Aperiodic
This is a single DT triangle. It occurs once and never repeats. So it cannot be periodic.

(d)   (1 pt) $g[n] = \sin\left(\dfrac{3n}{20}\right)$ Aperiodic
This is a DT sinusoid. Even though the CT function, $g(t) = \sin\left(\dfrac{3t}{20}\right)$, is periodic with period, $\dfrac{40\pi}{3}$, the DT function,

$g[n] = \sin\left(\dfrac{3n}{20}\right)$, which is a sampled version of the CT function, is not periodic because the sampling rate (1 sample per second) and the fundamental frequency of the CT sinusoid ($\dfrac{3}{40\pi}$) are not rationally related.

3. A DT system is excited by the unit sequence function and the response is $K(1-\alpha^n)$.

(a) (1 pt) If K is -2 and α is 1.1, is the system BIBO stable ?

No
The excitation (the unit sequence) is bounded. But the response, $-2(1-(1.1)^n)$ grows without bound as n increases.

(b) (1 pt) If K is 2 and α is -1.1, is the system BIBO stable?

No
The response, $2(1-(-1.1)^n)$ also grows without bound as n increases.

4. (4 pts) A DT function is $g[n] = 3\delta[n-4]\mathrm{ramp}[n+1]$. What is its only non-zero numerical value?

The only non-zero value occurs at $n = 4$ where the impulse has its only non-zero value, one. At that (discrete) time, the value of the function is $g[4] = 3(1)\mathrm{ramp}[5] = 15$.

5. Let $g[n] = u[n+3] - u[n-5]$. (Sketching these functions may help to find the answers.)

(a) (3 pts) What is the numerical sum of all the values of $g[n]$?

$g[n]$ consists of a sequence of unit DT impulses from $n = -3$ through $n = +4$, for a total of eight unit DT impulses. Therefore the sum is 8. There is no impulse at $n = 5$ because at that discrete time,
$g[5] = u[5+3] - u[5-5] = u[8] - u[0] = 0$

(b) (5 pts) If $h[n] = g[3n]$, what is the numerical value of the sum of all the values of $h[n]$?

The transformed function is $h[n] = g[3n] = u[3n + 3] - u[3n - 5]$.
The first unit sequence begins at $n = -1$ and the second one begins
at $n = 2$. So there are unit DT impulses in $h[n]$ at $n = -1, 0$ and 1
for a total of three. The numerical value is, therefore, 3.

6. (8 pts) Find the signal energy of the signal, $x(t) = \text{rect}(t) + \text{rect}\left(t - \dfrac{1}{2}\right)$.

(Sketching the function may help to find the answer.)

From a sketch of the square of this function we can write,

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 \, dt = \int_{-\frac{1}{2}}^{0} |1|^2 \, dt + \int_{0}^{\frac{1}{2}} |2|^2 \, dt + \int_{\frac{1}{2}}^{1} |1|^2 \, dt = \frac{1}{2} + 2 + \frac{1}{2} = 3$$

Alternate Solution:

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 \, dt = \int_{-\infty}^{\infty} \left| \text{rect}(t) + \text{rect}\left(t - \frac{1}{2}\right) \right|^2 \, dt = \int_{-\infty}^{\infty} \text{rect}^2(t) + \text{rect}^2\left(t - \frac{1}{2}\right) + 2\text{rect}(t)\text{rect}\left(t - \frac{1}{2}\right) \, dt$$

When a unit rectangle function is squared its area does not change because $0^2 = 0$ and
$1^2 = 1$. When the two different rectangle functions are multiplied the product is only non-
zero where both are non-zero. Therefore

$$E_x = \int_{-\infty}^{\infty} \text{rect}(t) \, dt + \int_{-\infty}^{\infty} \text{rect}\left(t - \frac{1}{2}\right) \, dt + \int_{-\infty}^{\infty} 2\text{rect}(t)\text{rect}\left(t - \frac{1}{2}\right) \, dt$$

$$E_x = \int_{-\frac{1}{2}}^{\frac{1}{2}} dt + \int_{0}^{1} dt + 2\int_{0}^{\frac{1}{2}} dt = 1 + 1 + 1 = 3$$

7. (5pts) A DT signal consists of the periodic alternating sequence,
$\cdots 4, -2, 4, -2, 4, -2, \cdots$. What is the average signal power of this signal?

This is a periodic DT signal. The fundamental period is 2. The signal
energy in one period is $4^2 + (-2)^2 = 20$. The average signal power is the
signal energy in one period divided by the period. The average signal
power is 10.

Using formulas, call the signal "x". Then

$$P_x = \frac{1}{N} \sum_{n=\langle N \rangle} |x[n]|^2 = \frac{1}{2} \sum_{n=\langle 2 \rangle} |x[n]|^2 = \frac{1}{2} \sum_{n=0}^{1} |x[n]|^2 = \frac{1}{2} \sum_{n=0}^{1} \left(4^2 + (-2)^2\right) = \frac{20}{2} = 10$$