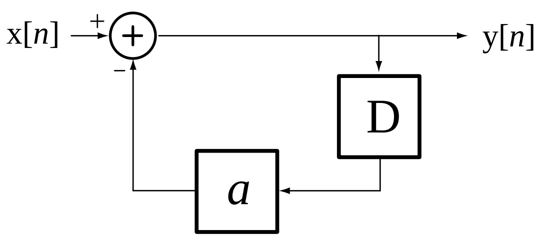
Solution of ECE 315 Test #2 Su03

1. (7 pts) A DT system is characterized by the block diagram below where x is the excitation and y is the response. Find its impulse response, $\mathrm{h}[n]$.

$\mathrm{h}[n] = (-a)^n \mathrm{u}[n]$

The difference equation is $\mathrm{y}[n] = \mathrm{x}[n] - a\,\mathrm{y}[n-1]$. The eigenvalue is *-a*. The impulse response is $\mathrm{h}[n] = K(-a)^n$. At time, $n = 0$, the system response to a unit impulse excitation is 1. Therefore, $K = 1$ and $\mathrm{h}[n] = (-a)^n \mathrm{u}[n]$.



Is this system stable?

If |*a*| < 1, it is stable. Otherwise it is unstable.

2. (4 pts) The impulse response of a DT system is zero for all negative time and, for $n \geq 0$, it is the alternating sequence, $1,-1,1,-1,1,-1,\cdots$ which continues forever. Is it stable?

The impulse response is not absolutely summable because the square is the sequence, $1,1,1,1,1,1,\cdots$ and the sum diverges.

No, the system is not stable.

OR

2. (4 pts) The impulse response of a DT system is zero for all negative time and, for $n \geq 0$, it is the alternating sequence, $1,-1,1,-1,1,-1$ followed by all zeros. Is it stable?

The impulse response is absolutely summable because the square is the sequence, $1,1,1,1,1,1,0,0,\cdots$ and the sum converges.

Yes, the system is stable.

3. (8 pts) Sketch the convolution, $\mathrm{y}[n] = \mathrm{x}[n] * \mathrm{h}[n]$, where $\mathrm{x}[n] = \mathrm{u}[n] - \mathrm{u}[n-a]$ and $\mathrm{h}[n] = \delta[n] - \delta[n-b]$. The sketch must include scales for the axes so that actual numerical values of $\mathrm{y}[n]$ at actual numerical values of discrete time, *n*, can be determined from the sketch.

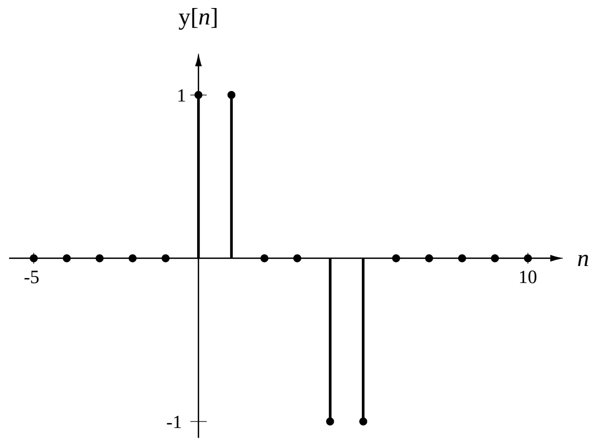
$$\mathrm{y}[n] = (\mathrm{u}[n] - \mathrm{u}[n-a]) * (\delta[n] - \delta[n-b])$$

$$\mathrm{y}[n] = \mathrm{u}[n] * (\delta[n] - \delta[n-b]) - \mathrm{u}[n-a] * (\delta[n] - \delta[n-b])$$

$$y[n] = u[n] * \delta[n] - u[n] * \delta[n-b] - u[n-a] * \delta[n] + u[n-a] * \delta[n-b]$$

$$y[n] = u[n] - u[n-b] - u[n-a] + u[n-(a+b)]$$

If $a = 4$ and $b = 2$,



4. (4 pts) Two systems have impulse responses, $h_1[n] = (0.9)^n u[n]$ and $h_2[n] = \delta[n] - (0.9)^n u[n]$. When these two systems are connected in parallel what is the response, $y[n]$, of the overall system to the excitation, $x[n] = u[n]$?

$y[n] = u[n]$

When connected in parallel the overall system impulse response is the sum of the two individual system impulse responses which is $h[n] = \delta[n]$. Therefore the response of the overall system to a unit sequence is the unit sequence.

5. (12 pts) Two systems have impulse responses, $h_1(t) = u(t) - u(t-a)$ and $h_2(t) = \text{rect}\left(\frac{t - \frac{a}{2}}{a}\right)$. If these two systems are connected in cascade, sketch the response, $y(t)$, of the overall system to the excitation, $x(t) = \delta(t)$. The sketch must include scales for the axes so that actual numerical values of $y(t)$ at actual numerical values of time, t, can be determined from the sketch.

The impulse response of the overall system is the convolution of the two impulse responses,

$$h(t) = [u(t) - u(t-a)] * \text{rect}\left(\frac{t - \frac{a}{2}}{a}\right) = [u(t) - u(t-a)] * [u(t) - u(t-a)]$$

$$h(t) = [u(t) - u(t-a)] * u(t) - [u(t) - u(t-a)] * u(t-a)$$

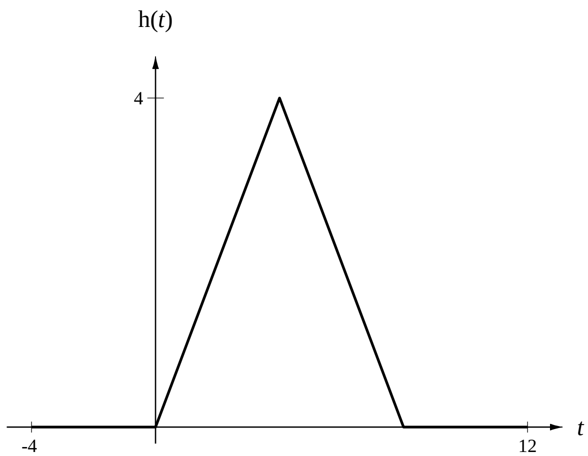
$$h(t) = u(t) * u(t) - u(t-a) * u(t) - u(t) * u(t-a) + u(t-a) * u(t-a)$$

Using $u(t) * u(t) = \text{ramp}(t)$,

$$h(t) = \text{ramp}(t) - \text{ramp}(t-a) - \text{ramp}(t-a) + \text{ramp}(t-2a)$$

$$h(t) = \text{ramp}(t) - 2\,\text{ramp}(t-a) + \text{ramp}(t-2a)$$

If $a = 4$

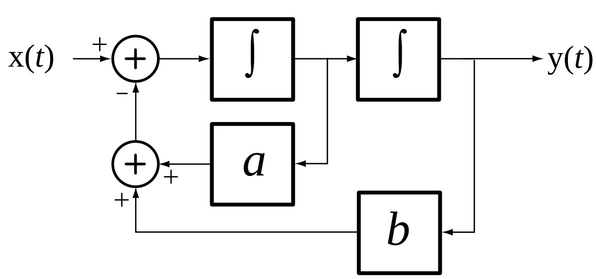


6. (8 pts) Write the differential equation for the system whose block diagram description is below. (Be sure to observe the signs on the summers.)

Differential Equation

$$y''(t) = x(t) - a\,y'(t) - b\,y(t) \quad \text{or} \quad y''(t) + a\,y'(t) + b\,y(t) = x(t)$$

The eigenvalues are $\dfrac{-a \pm \sqrt{a^2 - 4b}}{2}$.



Is this system stable?

If both eigenvalues have a real part less than zero, the system is stable. Otherwise it is unstable.