Solution to ECE 315 Test #3 F03

1. $y(t) = x^2(t)$

$$x_1(t) = g(t) \Rightarrow y_1(t) = g^2(t)$$
$$x_2(t) = g(t - t_0) \Rightarrow y_2(t) = g^2(t - t_0)$$
$$y_1(t - t_0) = g^2(t - t_0) = y_2(t) \Rightarrow \text{Time Invariant}$$

$$x_1(t) = g(t) \Rightarrow y_1(t) = g^2(t)$$
$$x_2(t) = K\,g(t) \Rightarrow y_2(t) = K^2 g^2(t) \neq K\,y_1(t) \Rightarrow \text{Not Homogeneous} \Rightarrow \text{Not Linear}$$

If any arbitrary real excitation, $x(t)$, is negated, the response does not change. Therefore unique excitations do not always produce unique responses. The system is not invertible

2. $y(t) = \int_{-\infty}^{t} x(\lambda)\,d\lambda$

If $x(t)$ is a finite constant (therefore bounded), $y(t)$, increases without bound. The system is not stable.

The present value of $y(t)$ depends on past values of $x(t)$ because the integral is for all time since negative infinity. Therefore the system has memory and is dynamic, not static.

3. $y''(t) = 2\,x(t) - 4\,y'(t) - 4\,y(t)$

The equation can be re-written as

$$\frac{1}{2}y''(t) + 2\,y'(t) + 2\,y(t) = x(t) \ .$$

This says that, given a knowledge of $y(t)$, $x(t)$ can be directly computed from $y(t)$ and its derivatives. The system is invertible.

The eigenvalues of the characteristic equation are -2 and -2 (a repeated root). Both of these have a negative real part. The system is stable

4. $y[n] = 2\,x[n] - 4\,y[n-1] - 4\,y[n-2]$

The response, y, at time, *n*, depends on the excitation, x, at the same time, *n* and the previous values of the response at the previous two discrete times, *n* - 1 and *n* - 2, and not on any future values of the excitation. The system is causal.

The current value of the response, y, at time *n* depends on prior values of y which depended on prior values of the excitation, x. Therefore the system has memory and is dynamic, not static.

5. $\mathrm{y}[n] = \cos(\mathrm{x}[n])$

$$\mathrm{x}_1[n] = \mathrm{g}[n] \Rightarrow \mathrm{y}_1[n] = \cos(\mathrm{g}[n])$$
$$\mathrm{x}_2[n] = \mathrm{g}[n - n_0] \Rightarrow \mathrm{y}_2[n] = \cos(\mathrm{g}[n - n_0])$$
$$\mathrm{y}_1[n - n_o] = \cos(\mathrm{g}[n - n_o]) = \mathrm{y}_2[n] \Rightarrow \text{Time Invariant}$$

$$\mathrm{x}_1[n] = \mathrm{g}[n] \Rightarrow \mathrm{y}_1[n] = \cos(\mathrm{g}[n])$$
$$\mathrm{x}_2[n] = K\,\mathrm{g}[n] \Rightarrow \mathrm{y}_2[n] = \cos(K\,\mathrm{g}[n - n_0]) \neq K\cos(\mathrm{g}[n - n_0]) \Rightarrow \text{Not Homogeneous} \Rightarrow \text{Not Linear}$$

$$\mathrm{x}[n] = \cos^{-1}(\mathrm{y}[n])$$

The inverse cosine function is multiple valued. Therefore the system is not invertible.