Solution to Test #4 ECE 315 F02

1. $a_0\,\mathrm{y}[n] + a_1\,\mathrm{y}[n-1] = \mathrm{x}[n]$

The eigenvalue is found from the characteristic equation, $a_0\alpha + a_1 = 0 \Rightarrow \alpha = -\frac{a_1}{a_0}$. If $|\alpha| \geq 1$ the system is unstable.

For an excitation, $\mathrm{x}_1[n] = \mathrm{g}[n]$ the response is $\mathrm{y}_1[n]$ and

$$a_0\,\mathrm{y}_1[n] + a_1\,\mathrm{y}_1[n-1] = \mathrm{g}[n]\ .$$

For an excitation, $\mathrm{x}_2[n] = \mathrm{g}[n-n_0]$, the response is $\mathrm{y}_2[n]$ and

$$a_0\,\mathrm{y}_2[n] + a_1\,\mathrm{y}_2[n-1] = \mathrm{g}[n-n_0]\ .$$

The first equation can be written as

$$a_0\,\mathrm{y}_1[n-n_0] + a_1\,\mathrm{y}_1[n-n_0-1] = \mathrm{g}[n-n_0]\ .$$

Therefore

$$a_0\,\mathrm{y}_2[n] + a_1\,\mathrm{y}_2[n-1] = a_0\,\mathrm{y}_1[n-n_0] + a_1\,\mathrm{y}_1[n-n_0-1]$$

and this equation can only be satisfied for all time if $\mathrm{y}_2[n] = \mathrm{y}_1[n-n_0]$. This proves time invariance.

2. $\mathrm{y}(t) = \exp(\mathrm{x}(t+2))$

This system is stable because if x is bounded so is y.

If the excitation, x, is multiplied by a constant, *K*, the response is raised to the *K*th power and, in general, that amounts to the response being multiplied by a different factor than the excitation . Therefore the system in inhomogeneous and therefore non-linear.

This system is non-causal because the response at time, *t*, depends on the excitation at a later time, $t+2$.

3. $a_2\,\mathrm{y}''(t) + a_1\,\mathrm{y}'(t) + a_0\,\mathrm{y}(t) = \mathrm{x}(t)$

The eigenvalues are the solutions of the characteristic equation, $a_2\alpha^2 + a_1\alpha + a_0 = 0$. If either of the eigenvalues has a real part greater than or equal to zero the system is unstable.

This is a linear differential equation with constant coefficients and the zero-input response is zero. Therefore the system is linear and time invariant.

4. $\mathrm{y}(t) = -|t\,\mathrm{x}(t)|$

If we apply the excitation, $\mathrm{x}_1(t) = \mathrm{g}(t)$ the response is $\mathrm{y}_1(t) = -|t\,\mathrm{g}(t)|$. If we apply the excitation, $\mathrm{x}_2(t) = \mathrm{g}(t - t_0)$ the response is $\mathrm{y}_2(t) = -|t\,\mathrm{g}(t - t_0)|$. If we delay the first response by t_0 we get $\mathrm{y}_1(t - t_0) = -|(t - t_0)\,\mathrm{g}(t - t_0)| \neq \mathrm{y}_2(t)$. Therefore the system is time variant.

If at some time $\mathrm{y}(t)$ has a certain value we don't know whether $t\,\mathrm{x}(t)$ is the negative of that value or the same as that value because taking the magnitude of $t\,\mathrm{x}(t)$ masks the effect of its sign. Therefore this system is non-invertible.