Solution to ECE 315 Test #3 F04

For each system determine whether it is BIBO Stable or BIBO Unstable, Static or Dynamic, Causal or Non-Causal, Linear or Non-Linear, Time Invariant or Time Variant. (In each case the excitation is x and the response is y.) Indicate the correct answer by circling it. A proof of your answer is not required, just the answer.

1.

$$\mathrm{y}(t) = \int_{-\infty}^{t} \mathrm{x}(\lambda) d\lambda$$

BIBO Unstable because if the exitation were a unit step (a bounded signal), the response would grow without bound.

Also we could differentiate both sides yielding $\mathrm{y}'(t) = \mathrm{x}(t)$. The eigenvalue for this differential equation is zero so its real part is not less than zero.

Dynamic because present values of y depend on former values of x through the integration process.

Causal because the present value of y depends only on past and present values of t, not on future values of t.

Linear. Multiplying any excitation by a constant simply multiplies the response by that same constant because a constant can be taken outside an integral. Therefore the system is homogeneous. Adding two excitations also adds their responses because an integral of a sum is the sum of the integrals. Therefore the system is additive.

Time Invariant. Shifting the excitation shifts the response by the same amount of time.

2.

$$\mathrm{y}[n] = |\mathrm{x}[n]|$$

BIBO Stable because for any bounded x, y is also bounded because it is the absolute value of x.

Static because the present value of y depends only on the present value of x.

Causal because it is static.

Non-Linear because if $\mathrm{x} = 1$, $\mathrm{y} = 1$ and if we multiply the excitation by -1, we multiply the response by $+1$, not -1. Not homogeneous and not linear.

Time Invariant because if we shift x in time we shift y by the same amount.

3.

$$\mathrm{y}(t) = \mathrm{x}\left(\frac{t}{2}\right) + 4$$

BIBO Stable because if x is bounded, y is simply some bounded value plus 4 and that sum is also bounded.

Dynamic because the present value of y depends on the value of x at some other time (except at $t = 0$).

Non-Causal because for any negative time, t, $t/2$ is in the future.

Non-Linear because if $x(\frac{t}{2}) = 1$, $y(t) = 5$ and we double x to 2, y does not double. Not homogeneous and not linear.

Time Variant. This is probably the trickiest proof. Let $x_1(t) = g(t)$. Then$y_1(t) = g(\frac{t}{2}) + 4$. Let $x_2(t) = g(t - t_0)$. Then $y_2(t) = g(\frac{t}{2} - t_0) + 4$.

$y_1(t - t_0) = g(\frac{t-t_0}{2}) + 4 \neq y_2(t)$.

4.

$$2y'(t) - y(t) = x(t)$$

BIBO Unstable because the eigenvalue is 1/2 whose real part is greater than zero. (If the sign of the second term on the left is changed, the system becomes stable because the eigenvalue becomes -1/2.)

Dynamic because this system can be represented by a block diagram with an integrator in it and an integrator has memory. Looking at it another way, in order to have the derivative of y in the system the system must have some knowledge of the past history of y. Imagine that at some instant of time you know what $y(t)$ is. What is $y\prime(t)$? It is impossible to say without knowing how the y function is moving at that point and that requires memory.

Causal because the present value of y depends only on present and past values of x.

Linear and Time Invariant by the usual proof methods for this very common type of system.