Solution of ECE 315 Test 4 F06

In each system description below the excitation is x and the response is y.
Classify each system by circling the correct properties.

1. $\mathrm{y}[n] = \mathrm{x}^2[n]$

$\mathrm{x}_1[n] = \mathrm{g}[n] \Rightarrow \mathrm{y}_1[n] = \mathrm{g}^2[n]$, $\mathrm{x}_2[n] = K\,\mathrm{g}[n] \Rightarrow \mathrm{y}_2[n] = (K\,\mathrm{g}[n])^2 \neq K\,\mathrm{y}_1[n] \Rightarrow$ Non-Linear

$\mathrm{x}_1[n] = \mathrm{g}[n] \Rightarrow \mathrm{y}_1[n] = \mathrm{g}^2[n]$, $\mathrm{x}_2[n] = \mathrm{g}[n-n_0] \Rightarrow \mathrm{y}_2[n] = \mathrm{g}^2[n-n_0] = \mathrm{y}_1[n-n_0] \Rightarrow$ Time Invariant

x bounded $\Rightarrow$ x^2 bounded $\Rightarrow$ y bounded $\Rightarrow$ Stable

$\mathrm{y}[n]$ depends only on $\mathrm{x}[n] \Rightarrow$ Causal

$\mathrm{y}[n]$ depends only on $\mathrm{x}[n] \Rightarrow$ Static

$\mathrm{x}[n] = \pm\sqrt{\mathrm{y}[n]} \Rightarrow$ Non-Invertible

2. $2\mathrm{y}'(t) - \mathrm{y}(t) = \mathrm{x}(t)$

$2\mathrm{y}_1'(t) - \mathrm{y}_1(t) = \mathrm{g}(t) \Rightarrow 2K\,\mathrm{y}_1'(t) - K\,\mathrm{y}_1(t) = K\,\mathrm{g}(t)$

$2\mathrm{y}_2'(t) - \mathrm{y}_2(t) = K\,\mathrm{g}(t) \Rightarrow 2K\,\mathrm{y}_1'(t) - K\,\mathrm{y}_1(t) = 2\mathrm{y}_2'(t) - \mathrm{y}_2(t) \Rightarrow \mathrm{y}_2(t) = K\,\mathrm{y}_1(t) \Rightarrow$ Homogeneous

$2\mathrm{y}_1(t) - \mathrm{y}_1(t) = \mathrm{g}(t)$ and $2\mathrm{y}_2(t) - \mathrm{y}_2(t) = \mathrm{h}(t) \Rightarrow 2\mathrm{y}_1'(t) + 2\mathrm{y}_2'(t) - \mathrm{y}_1(t) - \mathrm{y}_2(t) = \mathrm{g}(t) + \mathrm{h}(t)$

$2\mathrm{y}_3'(t) - \mathrm{y}_3(t) = \mathrm{g}(t) + \mathrm{h}(t) \Rightarrow 2\mathrm{y}_1'(t) + 2\mathrm{y}_2'(t) - \mathrm{y}_1(t) - \mathrm{y}_2(t) = 2\mathrm{y}_3'(t) - \mathrm{y}_3(t) \Rightarrow \mathrm{y}_3(t) = \mathrm{y}_1(t) + \mathrm{y}_2(t)$

$\Rightarrow$ Additive $\Rightarrow$ Linear

$2\mathrm{y}_1'(t) - \mathrm{y}_1(t) = \mathrm{g}(t) \Rightarrow 2\mathrm{y}_1'(t-t_0) - \mathrm{y}_1(t-t_0) = \mathrm{g}(t-t_0)$

$2\mathrm{y}_2'(t) - \mathrm{y}_2(t) = \mathrm{g}(t-t_0) \Rightarrow 2\mathrm{y}_1'(t-t_0) - \mathrm{y}_1(t-t_0) = 2\mathrm{y}_2'(t) - \mathrm{y}_2(t) \Rightarrow \mathrm{y}_2(t) = \mathrm{y}_1(t-t_0) \Rightarrow$ Time Invariant

Eigenvalue is +1/2 $\Rightarrow$ Unstable

$\int_{-\infty}^{t} [2\mathrm{y}'(\tau) - \mathrm{y}(\tau)]d\tau = \int_{-\infty}^{t} \mathrm{x}(\tau)d\tau \Rightarrow 2\mathrm{y}(t) = \int_{-\infty}^{t} \mathrm{x}(\tau)d\tau + \int_{-\infty}^{t} \mathrm{y}(\tau)d\tau \Rightarrow$ Causal and Dynamic

$\mathrm{x}(t) = 2\mathrm{y}'(t) - \mathrm{y}(t) \Rightarrow$ Invertible

3. $2\mathrm{y}[n-1] - \mathrm{y}[n-2] = \mathrm{x}[n]$

$2\mathrm{y}_1[n-1] - \mathrm{y}_1[n-2] = \mathrm{g}[n] \Rightarrow 2K\,\mathrm{y}_1[n-1] - K\,\mathrm{y}_1[n-2] = K\,\mathrm{g}[n]$

$2\mathrm{y}_2[n-1] - \mathrm{y}_2[n-2] = K\,\mathrm{g}[n] \Rightarrow 2K\,\mathrm{y}_1[n-1] - K\,\mathrm{y}_1[n-2] = 2\mathrm{y}_2[n-1] - \mathrm{y}_2[n-2] \Rightarrow \mathrm{y}_2[n] = K\,\mathrm{y}_1[n]$

$2\mathrm{y}_1[n-1] - \mathrm{y}_1[n-2] = \mathrm{g}[n]$ and $2\mathrm{y}_2[n-1] - \mathrm{y}_2[n-2] = \mathrm{h}[n] \Rightarrow 2\mathrm{y}_1[n-1] + 2\mathrm{y}_2[n-1] - \mathrm{y}_1[n-2] - \mathrm{y}_2[n-2] = \mathrm{g}[n] + \mathrm{h}[n]$

$2\mathrm{y}_3[n-1] - \mathrm{y}_3[n] = \mathrm{g}[n] + \mathrm{h}[n] \Rightarrow 2\mathrm{y}_1[n-1] + 2\mathrm{y}_2[n-1] - \mathrm{y}_1[n-2] - \mathrm{y}_2[n-2] = 2\mathrm{y}_3[n-1] - \mathrm{y}_3[n]$

Linear

$$2\,\mathrm{y}_1[n-1]-\mathrm{y}_1[n-2]=\mathrm{g}[n]\Rightarrow 2\,\mathrm{y}_1[n-n_0-1]-\mathrm{y}_1[n-n_0-2]=\mathrm{g}[n-n_0]$$

$$2\,\mathrm{y}_2[n-1]-\mathrm{y}_2[n-2]=\mathrm{g}[n-n_0]\Rightarrow 2\,\mathrm{y}_1[n-n_0-1]-\mathrm{y}_1[n-n_0-2]=2\,\mathrm{y}_2[n-1]-\mathrm{y}_2[n-2]\Rightarrow$$

$$\mathrm{y}_2[n]=\mathrm{y}_1(n-n_0)\Rightarrow \text{Time Invariant}$$

Eigenvalue is +1/2 $\Rightarrow$ Stable

$\mathrm{y}[n-1]$ depends on $\mathrm{x}[n]$ $\Rightarrow$ Non-Causal

$\mathrm{y}[n-1]$ depends on $\mathrm{x}[n]$ and $\mathrm{y}[n-2]$ $\Rightarrow$ Dynamic

$\mathrm{x}[n]=2\,\mathrm{y}[n-1]-\mathrm{y}[n-2]\Rightarrow$ Invertible

4. $\mathrm{y}[n]=\left(n^2+1\right)\left(\mathrm{x}[n]-\mathrm{x}[n-1]\right)$

$$2\,\mathrm{y}_1[n]=\left(n^2+1\right)\left(\mathrm{g}[n]-\mathrm{g}[n-1]\right)\Rightarrow 2K\,\mathrm{y}_1[n]=\left(n^2+1\right)\left(K\,\mathrm{g}[n]-K\,\mathrm{g}[n-1]\right)$$

$$2\,\mathrm{y}_2[n]=\left(n^2+1\right)\left(K\,\mathrm{g}[n]-K\,\mathrm{g}[n-1]\right)\Rightarrow 2K\,\mathrm{y}_1[n]=2\,\mathrm{y}_2[n]\Rightarrow \text{Homogeneous}$$

$$2\,\mathrm{y}_1[n]=\left(n^2+1\right)\left(\mathrm{g}[n]-\mathrm{g}(n-1)\right)\text{ and }2\,\mathrm{y}_2[n]=\left(n^2+1\right)\left(\mathrm{h}[n]-\mathrm{h}(n-1)\right)\Rightarrow$$

$$2\,\mathrm{y}_1[n]+2\,\mathrm{y}_2[n]=\left(n^2+1\right)\left(\mathrm{g}[n]-\mathrm{g}(n-1)\right)+\left(n^2+1\right)\left(\mathrm{h}[n]-\mathrm{h}(n-1)\right)$$

$$2\,\mathrm{y}_3[n]=\left(n^2+1\right)\left(\mathrm{g}[n]+\mathrm{h}[n]-\mathrm{g}(n-1)-\mathrm{h}[n-1]\right)\Rightarrow 2\,\mathrm{y}_1[n]+2\,\mathrm{y}_2[n]=2\,\mathrm{y}_3[n]\Rightarrow \text{Additive}\Rightarrow\text{Linear}$$

$$2\,\mathrm{y}_1[n]=\left(n^2+1\right)\left(\mathrm{g}[n]-\mathrm{g}[n-1]\right)\Rightarrow 2\,\mathrm{y}_1[n-n_0]=\left((n-n_0)^2+1\right)\left(\mathrm{g}[n-n_0]-\mathrm{g}[n-n_0-1]\right)$$

$$2\,\mathrm{y}_2[n]=\left(n^2+1\right)\left(\mathrm{g}[n-n_0]-\mathrm{g}[n-n_0-1]\right)\neq 2\,\mathrm{y}_1[n-n_0]\Rightarrow \text{Time Variant}$$

$\mathrm{y}[n]$ depends on $\mathrm{x}[n]$ and $\mathrm{x}[n-1]$ $\Rightarrow$ Causal

As $n\to\pm\infty$, $\mathrm{y}[n]\to\pm\infty$ if $\mathrm{x}[n]-\mathrm{x}[n-1]\neq 0$

$\mathrm{y}[n]$ depends on $\mathrm{x}[n]$ and $\mathrm{x}[n-1]$ $\Rightarrow$ Dynamic

$$\frac{\mathrm{y}[n]}{n^2+1}=\mathrm{x}[n]-\mathrm{x}[n-1]\Rightarrow\sum_{m=-\infty}^{n}\frac{\mathrm{y}[m]}{\left(m^2+1\right)}=\sum_{m=-\infty}^{n}\left(\mathrm{x}[m]-\mathrm{x}[m-1]\right)$$

$$\sum_{m=-\infty}^{n}\frac{\mathrm{y}[m]}{\left(m^2+1\right)}=\sum_{m=-\infty}^{n}\mathrm{x}[m]-\sum_{m=-\infty}^{n}\mathrm{x}[m-1]=\mathrm{x}[n]\Rightarrow \text{Invertible}$$

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In each system description below the excitation is x and the response is y.
Classify each system by circling the correct properties.

1. $y[n] = \sqrt{x[n]}$

$x_1[n] = g[n] \Rightarrow y_1[n] = \sqrt{g[n]}$, $x_2[n] = K\,g[n] \Rightarrow y_2[n] = \sqrt{K\,g[n]} \neq K\,y_1[n] \Rightarrow$ Not Homogeneous $\Rightarrow$ Non-Linear

$x_1[n] = g[n] \Rightarrow y_1[n] = \sqrt{g[n]}$, $x_2[n] = g[n-n_0] \Rightarrow y_2[n] = \sqrt{g[n-n_0]} = y_1[n-n_0] \Rightarrow$ Time Invariant

x bounded $\Rightarrow \sqrt{x}$ bounded $\Rightarrow$ y bounded $\Rightarrow$ Stable

$y[n]$ depends only on $x[n] \Rightarrow$ Causal

$y[n]$ depends only on $x[n] \Rightarrow$ Static

$x[n] = y^2[n] \Rightarrow$ Invertible

2. $y'(t) + 2\,y(t) = x(t)$

$y_1'(t) + 2\,y_1(t) = g(t) \Rightarrow K\,y_1'(t) + 2K\,y_1(t) = K\,g(t)$

$y_2'(t) + 2\,y_2(t) = K\,g(t) \Rightarrow 2\,y_1'(t) + K\,y_1(t) = 2\,y_2'(t) + y_2(t) \Rightarrow y_2(t) = K\,y_1(t) \Rightarrow$ Homogeneous

$y_1(t) + 2\,y_1(t) = g(t)$ and $y_2(t) + 2\,y_2(t) = h(t) \Rightarrow y_1'(t) + 2\,y_2'(t) + y_1(t) + 2\,y_2(t) = g(t) + h(t)$

$y_3'(t) + 2\,y_3(t) = g(t) + h(t) \Rightarrow y_1'(t) + y_2'(t) + 2\,y_1(t) + 2\,y_2(t) = y_3'(t) + 2\,y_3(t) \Rightarrow y_3(t) = y_1(t) + y_2(t)$

$\Rightarrow$ Additive $\Rightarrow$ Linear

$y_1'(t) + 2\,y_1(t) = g(t) \Rightarrow y_1'(t-t_0) + 2\,y_1(t-t_0) = g(t-t_0)$

$y_2'(t) + 2\,y_2(t) = g(t-t_0) \Rightarrow y_1'(t-t_0) + 2\,y_1(t-t_0) = y_2'(t) + 2\,y_2(t) \Rightarrow y_2(t) = y_1(t-t_0) \Rightarrow$ Time Invariant

Eigenvalue is -2 $\Rightarrow$ Stable

$$\int_{-\infty}^{t} [y'(\tau) + 2\,y(\tau)]\,d\tau = \int_{-\infty}^{t} x(\tau)\,d\tau \Rightarrow y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau - 2\int_{-\infty}^{t} y(\tau)\,d\tau \Rightarrow \text{Causal and Dynamic}$$

$x(t) = y'(t) + 2\,y(t) \Rightarrow$ Invertible

3. $y[n-1] + 2\,y[n-2] = x[n]$

$y_1[n-1] + 2\,y_1[n-2] = g[n] \Rightarrow K\,y_1[n-1] + 2K\,y_1[n-2] = K\,g[n]$

$y_2[n-1] + 2\,y_2[n-2] = K\,g[n] \Rightarrow K\,y_1[n-1] + 2K\,y_1[n-2] = y_2[n-1] + 2\,y_2[n-2] \Rightarrow y_2[n] = K\,y_1[n]$

$y_1[n-1] + 2\,y_1[n-2] = g[n]$ and $y_2[n-1] + 2\,y_2[n-2] = h[n] \Rightarrow y_1[n-1] + y_2[n-1] + 2\,y_1[n-2] + 2\,y_2[n-2] = g[n] + h[n]$

$y_3[n-1] + 2\,y_3[n] = g[n] + h[n] \Rightarrow y_1[n-1] + y_2[n-1] + 2\,y_1[n-2] + 2\,y_2[n-2] = y_3[n-1] + 2\,y_3[n]$

Linear

$y_1[n-1]+2y_1[n-2]=g[n] \Rightarrow y_1[n-n_0-1]+2y_1[n-n_0-2]=g[n-n_0]$

$y_2[n-1]+2y_2[n-2]=g[n-n_0] \Rightarrow y_1[n-n_0-1]+2y_1[n-n_0-2]=y_2[n-1]+2y_2[n-2] \Rightarrow$

$y_2[n]=y_1(n-n_0) \Rightarrow$ Time Invariant

Eigenvalue is -2 $\Rightarrow$ Unstable

$y[n-1]$ depends on $x[n]$ $\Rightarrow$ Non-Causal

$y[n-1]$ depends on $x[n]$ and $y[n-2]$ $\Rightarrow$ Dynamic

$x[n]=y[n-1]+2y[n-2] \Rightarrow$ Invertible

4. $y[n]=(n^2+1)(x[n]-x[n-1])$

$2y_1[n]=(n^2+1)(g[n]-g[n-1]) \Rightarrow 2Ky_1[n]=(n^2+1)(Kg[n]-Kg[n-1])$

$2y_2[n]=(n^2+1)(Kg[n]-Kg[n-1]) \Rightarrow 2Ky_1[n]=2y_2[n] \Rightarrow$ Homogeneous

$2y_1[n]=(n^2+1)(g[n]-g(n-1))$ and $2y_2[n]=(n^2+1)(h[n]-h(n-1)) \Rightarrow$

$2y_1[n]+2y_2[n]=(n^2+1)(g[n]-g(n-1))+(n^2+1)(h[n]-h(n-1))$

$2y_3[n]=(n^2+1)(g[n]+h[n]-g(n-1)-h[n-1]) \Rightarrow 2y_1[n]+2y_2[n]=2y_3[n] \Rightarrow$ Additive $\Rightarrow$ Linear

$2y_1[n]=(n^2+1)(g[n]-g[n-1]) \Rightarrow 2y_1[n-n_0]=((n-n_0)^2+1)(g[n-n_0]-g[n-n_0-1])$

$2y_2[n]=(n^2+1)(g[n-n_0]-g[n-n_0-1]) \neq 2y_1[n-n_0] \Rightarrow$ Time Variant

$y[n]$ depends on $x[n]$ and $x[n-1]$ $\Rightarrow$ Causal

As $n \to \pm\infty$, $y[n] \to \pm\infty$ if $x[n]-x[n-1] \neq 0 \Rightarrow$ Unstable

$y[n]$ depends on $x[n]$ and $x[n-1]$ $\Rightarrow$ Dynamic

$$\frac{y[n]}{n^2+1}=x[n]-x[n-1] \Rightarrow \sum_{m=-\infty}^{n} \frac{y[m]}{(m^2+1)} = \sum_{m=-\infty}^{n} (x[m]-x[m-1])$$

$$\sum_{m=-\infty}^{n} \frac{y[m]}{(m^2+1)} = \sum_{m=-\infty}^{n} x[m] - \sum_{m=-\infty}^{n} x[m-1] = x[n] \Rightarrow \text{Invertible}$$