Solution of ECE 315 Test 3 F07

1. $\mathrm{y}(t) = \mathrm{ramp}(\mathrm{x}(t))$ Linear **Non-Linear** **Static** Dynamic **Stable** Unstable
Invertible **Non-Invertible** **Time Invariant** Time Variant

$\mathrm{x}_1(t) = \mathrm{g}(t) \Rightarrow \mathrm{y}_1(t) = \mathrm{ramp}(\mathrm{g}(t))$, $\mathrm{x}_2(t) = K\,\mathrm{g}(t) \Rightarrow \mathrm{y}_2(t) = \mathrm{ramp}(K\,\mathrm{g}(t))$

Let $K = -1$. Then $\mathrm{y}_2(t) = \mathrm{ramp}(-\mathrm{g}(t))$ and $K\mathrm{y}_1(t) = -\mathrm{y}_1(t) \neq \mathrm{y}_2(t)$ unless $\mathrm{g}(t) = 0$

Not Homogeneous. Non-Linear.
y at any time *t* depends only on x at the same time. Static
Any finite x produces a finite y. Stable
Any negative x produces $\mathrm{y} = 0$. Therefore if $\mathrm{y} = 0$, it is impossible to determine x. Non-Invertible

$\mathrm{x}_1(t) = \mathrm{g}(t) \Rightarrow \mathrm{y}_1(t) = \mathrm{ramp}(\mathrm{g}(t))$, $\mathrm{x}_2(t) = \mathrm{g}(t - t_0) \Rightarrow \mathrm{y}_2(t) = \mathrm{ramp}(\mathrm{g}(t - t_0)) = \mathrm{y}_1(t - t_0)$ Time Invariant

2. $\mathrm{y}[n] = 3\mathrm{x}[n] - 2\mathrm{x}[n+1]$ **Linear** Non-Linear Static **Dynamic** **Stable** Unstable
Causal **Non-Causal** **Time Invariant** Time Variant

$\mathrm{x}_1[n] = \mathrm{g}[n] \Rightarrow \mathrm{y}_1[n] = 3\mathrm{g}[n] - 2\mathrm{g}[n+1]$, $\mathrm{x}_2[n] = K\,\mathrm{g}[n] \Rightarrow \mathrm{y}_2[n] = 3K\,\mathrm{g}[n] - 2K\,\mathrm{g}[n+1]$

$K\,\mathrm{y}_1[n] = 3K\,\mathrm{g}[n] - 2K\,\mathrm{g}[n+1] = \mathrm{y}_2[n]$ Homogeneous

$\mathrm{x}_1[n] = \mathrm{g}[n] \Rightarrow \mathrm{y}_1[n] = 3\mathrm{g}[n] - 2\mathrm{g}[n+1]$, $\mathrm{x}_2[n] = \mathrm{h}[n] \Rightarrow \mathrm{y}_2[n] = 3\mathrm{h}[n] - 2\mathrm{h}[n+1]$

$\mathrm{x}_3[n] = \mathrm{g}[n] + \mathrm{h}[n] \Rightarrow \mathrm{y}_3[n] = 3(\mathrm{g}[n] + \mathrm{h}[n]) - 2(\mathrm{g}[n+1] + \mathrm{h}[n+1]) = \mathrm{y}_1[n] + \mathrm{y}_2[n]$ Additive and Linear

y at time *n* depends on x at time $n+1$ (in the future) Non-Causal

$\mathrm{x}_1[n] = \mathrm{g}[n] \Rightarrow \mathrm{y}_1[n] = 3\mathrm{g}[n] - 2\mathrm{g}[n+1]$, $\mathrm{x}_2[n] = \mathrm{g}[n - n_0] \Rightarrow \mathrm{y}_2[n] = 3\mathrm{g}[n - n_0] - 2\mathrm{g}[n - n_0 + 1]$

$\mathrm{y}_1[n - n_0] = 3\mathrm{g}[n - n_0] - 2\mathrm{g}[n - n_0 + 1] = \mathrm{y}_2[n]$ Time Invariant

3. $\frac{d}{dt}(\mathrm{y}(t)) = \mathrm{x}(t)$ **Linear** Non-Linear Static **Dynamic** Stable **Unstable**
Invertible Non-Invertible **Time Invariant** Time Variant

$\mathrm{x}_1(t) = \mathrm{g}(t) \Rightarrow \frac{d}{dt}(\mathrm{y}_1(t)) = \mathrm{g}(t)$, $\mathrm{x}_2(t) = K\,\mathrm{g}(t) \Rightarrow \frac{d}{dt}(\mathrm{y}_2(t)) = K\,\mathrm{g}(t)$

$K\frac{d}{dt}(\mathrm{y}_1(t)) = K\,\mathrm{g}(t) \Rightarrow K\frac{d}{dt}(\mathrm{y}_1(t)) = \frac{d}{dt}(\mathrm{y}_2(t)) \Rightarrow \mathrm{y}_2(t) = K\,\mathrm{y}_1(t)$ Homogeneous

$\mathrm{x}_1(t) = \mathrm{g}(t) \Rightarrow \frac{d}{dt}(\mathrm{y}_1(t)) = \mathrm{g}(t)$, $\mathrm{x}_2(t) = \mathrm{h}(t) \Rightarrow \frac{d}{dt}(\mathrm{y}_2(t)) = \mathrm{h}(t)$

$\frac{d}{dt}(\mathrm{y}_1(t)) + \frac{d}{dt}(\mathrm{y}_2(t)) = \mathrm{g}(t) + \mathrm{h}(t) = \frac{d}{dt}(\mathrm{y}_3(t)) \Rightarrow \mathrm{y}_3(t) = \mathrm{y}_1(t) + \mathrm{y}_2(t)$ Additive and Linear

$\mathrm{y}(t) = \int_{-\infty}^{t} \mathrm{x}(\tau)\,d\tau$. Integration implies memory. Dynamic.

The eigenvalue is zero, whose real part is not negative. Unstable

The equation $\frac{d}{dt}(\mathrm{y}(t)) = \mathrm{x}(t)$ says in words that if y is known x can be found by differentiating y. Invertible

$\mathrm{x}_1(t) = \mathrm{g}(t) \Rightarrow \frac{d}{dt}(\mathrm{y}_1(t)) = \mathrm{g}(t)$, $\mathrm{x}_2(t) = \mathrm{g}(t - t_0) \Rightarrow \frac{d}{dt}(\mathrm{y}_2(t)) = \mathrm{g}(t - t_0)$

$\frac{d}{dt}(\mathrm{y}_1(t - t_0)) = \mathrm{g}(t - t_0) = \frac{d}{dt}(\mathrm{y}_2(t)) \Rightarrow \mathrm{y}_1(t - t_0) = \mathrm{y}_2(t)$ Time Invariant

4. $\mathrm{y}[n] = 4\mathrm{x}[3(n-1)]$ **Linear** Non-Linear Static **Dynamic** **Stable** Unstable
Causal **Non-Causal** Time Invariant **Time Variant**

$x_1[n] = g[n] \Rightarrow y_1[n] = 4g[3(n-1)]$, $x_2[n] = K\,g[n] \Rightarrow y_2[n] = 4K\,g[3(n-1)] = K\,y_1[n]$ Homogeneous

$x_1[n] = g[n] \Rightarrow y_1[n] = 4g[3(n-1)]$, $x_2[n] = h[n] \Rightarrow y_2[n] = 4h[3(n-1)]$

$x_3[n] = g[n] + h[n] \Rightarrow y_3[n] = 4(g[3(n-1)] + h[3(n-1)]) = y_1[n] + y_2[n]$ Additive and Linear

y at time $n = 5$ depends on x at time $3(5-1) = 12$ (in the future) Non-Causal

$x_1[n] = g[n] \Rightarrow y_1[n] = 4g[3(n-1)]$, $x_2[n] = g[n-n_0] \Rightarrow y_2[n] = 4g[3(n-1)-n_0]$

$y_1[n-n_0] = 4g[3(n-n_0-1)] \neq y_2[n]$ Time Variant

5. $y(t) = \begin{cases} 5x(t) & , \ |x(t)| \le 15 \\ 75 & , \ |x(t)| > 15 \end{cases}$ Linear **Non-Linear** **Static** Dynamic **Stable** Unstable

Invertible **Non-Invertible** **Time Invariant** Time Variant

$x_1(t) = g(t) \Rightarrow y_1(t) = \begin{cases} 5g(t) & , \ |g(t)| \le 15 \\ 75 & , \ |g(t)| > 15 \end{cases}$, $x_2(t) = K\,g(t) \Rightarrow y_2(t) = \begin{cases} 5g(t) & , \ |K\,g(t)| \le 15 \\ 75 & , \ |K\,g(t)| > 15 \end{cases}$

Let $K = 10$. Then if, at some time *t*, g = 10 , $K\,y_1 = 500$ but , *K*g = 100 and $y_2 = 75$.
Not Homogeneous. Non-Linear.
y at any time *t* depends only on x at the same time. Static
Any finite x produces a finite y. Stable
Any $|x(t)| > 15$ produces $y = 75$. Therefore if $y = 75$, it is impossible to determine x. Non-Invertible

$x_1(t) = g(t) \Rightarrow y_1(t) = \begin{cases} 5g(t) & , \ |g(t)| \le 15 \\ 75 & , \ |g(t)| > 15 \end{cases}$, $x_2(t) = g(t-t_0) \Rightarrow y_2(t) = \begin{cases} 5g(t-t_0) & , \ |K\,g(t-t_0)| \le 15 \\ 75 & , \ |K\,g(t-t_0)| > 15 \end{cases}$

$y_1(t-t_0) = \begin{cases} 5g(t-t_0) & , \ |g(t-t_0)| \le 15 \\ 75 & , \ |g(t-t_0)| > 15 \end{cases} = y_2(t)$ Time Invariant