Solution of ECE 316 Test #2 S04

1. (3 pts) What are the three signal processing steps in the derivation of the DFT that convert the transform pair, $x(t) \xleftrightarrow{\mathcal{F}} X(f)$ into the transform pair, $x[n] \xleftrightarrow{\mathcal{DFT}} X[k]$?

sampling (time sampling) windowing periodic repetition (frequency sampling)

2. A signal is sampled and the set of samples, $\{x[0],\cdots,x[7]\}$, is transformed, using the DFT, into the set of numbers, $\{X[0],\cdots,X[7]\} = \{6, 1-j2, -8, -3+j2, -2, -3-j2, -8, 1+j2\}$.

(a) (3 pts) What is the numerical average of the 8 numbers, $\{x[0],x[1],\cdots,x[7]\}$? 0.75

$$X[k] = \sum_{n=0}^{N_F-1} x[n] e^{-j2\pi\frac{nk}{N_F}} \Rightarrow X[0] = \sum_{n=0}^{7} x[n] \underbrace{e^0}_{1} \Rightarrow \frac{X[0]}{8} = \underbrace{\frac{1}{8}\sum_{n=0}^{7} x[n]}_{\text{average of the x's}} = \frac{6}{8} = \frac{3}{4} = 0.75$$

(b) (2 pts) What is the numerical value of $X[-1]$? $1+j2$

The DFT is periodic with period, N_F. In this case, $N_F = 8$. Therefore

$$X[-1] = X[-1+8] = X[7] = 1+j2$$

3. (2 pts) A real-valued signal is sampled and the set of samples is transformed, using the DFT, into another set of numbers, $\{X[0],X[1],\cdots,X[N-1]\}$. Two of these numbers are guaranteed to be real numbers. Which ones? $X[0]$ and $X\left[\frac{N}{2}\right]$

$$X[k] = \sum_{n=0}^{N_F-1} x[n] e^{-j2\pi\frac{nk}{N_F}} \Rightarrow X[0] = \sum_{n=0}^{N_F-1} \underbrace{x[n]}_{\text{real}}$$

$$\Rightarrow X\left[\frac{N}{2}\right] = \sum_{n=0}^{N_F-1} \underbrace{x[n]}_{\text{real}} \underbrace{e^{-j\pi n}}_{(-1)^n,\text{real}}$$

4. (4 pts) A signal, $x(t)$, is sampled 4 times and the samples are $\{x[0],x[1],x[2],x[3]\}$. Its DFT is $\{X[0],X[1],X[2],X[3]\}$. $X[3]$ can be written as $X[3] = a\,x[0] + b\,x[1] + c\,x[2] + d\,x[3]$. What are the numerical values of *a*,*b*,*c* and *d*?

$a=1 \quad b=j \quad c=-1 \quad d=-j$

$$X[k] = \sum_{n=0}^{N_F-1} x[n] e^{-j2\pi\frac{nk}{N_F}} \Rightarrow X[3] = \sum_{n=0}^{3} x[n] e^{-j\frac{3\pi}{2}n} = x[0] + j\,x[1] - x[2] - j\,x[3]$$