

## Solution of ECE 316 Test 3 S06

A set of samples  $\{x[0], x[1], x[2], x[3]\} = \{a, b, c, d\}$  is taken from a signal. A computer computes one period of the DFT of this set of samples as  $\{X[0], X[1], X[2], X[3]\} = \{A, B, C, D\}$ .

- (a) Find the value of  $X[1]$  (not  $x[1]$ ) as a linear combination of  $a, b, c$  and  $d$ .

$$X[1] = \underline{a - jb - c + jd}$$

$$X[k] = \sum_{n=0}^{N_F-1} x[n] e^{-j2\pi nk/N_F} \Rightarrow X[1] = \sum_{n=0}^3 x[n] e^{-j\pi n/2} = a - jb - c + jd$$

- (b) Find the value of  $X[-9]$  (not  $x[-9]$ ) as a linear combination of  $a, b, c$  and  $d$ .

$$X[-9] = \underline{a + jb - c - jd}$$

$$X[-9] = X[-9 + 12] = X[3] \Rightarrow X[3] = \sum_{n=0}^3 x[n] e^{-j3\pi n/2} = a + jb - c - jd$$

- (c) If the original set of samples was obtained by properly sampling a bandlimited periodic signal above its Nyquist rate over exactly one fundamental period, what is the average value of that signal as a linear combination of  $A, B, C$  and  $D$ ?

$$\text{Average value} = \underline{A/4}$$

$$\text{Average value} = \overline{X[0]}/4 = A/4$$

- (d) If  $\{X[0], X[1], X[2], X[3]\} = \{4, 2 + j3, -1, 2 - j3\}$  what is the numerical value of

$$X[-1]? \quad X[-1] = \underline{2 - j3}$$

$$X[-1] = X[-1 + 4] = X[3] = 2 - j3$$

- (e) (3 pts) If  $X[3] = 1 + j$  what is the numerical value of  $X[-3]$ ?

$$X[-3] = \underline{1 - j}$$

$$X[k] = X^*[-k] \Rightarrow X[-3] = X^*[3] = 1 - j$$

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A set of samples  $\{x[0], x[1], x[2], x[3]\} = \{a, b, c, d\}$  is taken from a signal. A computer computes one period of the DFT of this set of samples as  $\{X[0], X[1], X[2], X[3]\} = \{A, B, C, D\}$ .

- (a) Find the value of  $X[2]$  (not  $x[2]$ ) as a linear combination of  $a, b, c$  and  $d$ .

$$X[2] = \underline{a - b + c - d}$$

$$X[k] = \sum_{n=0}^{N_F-1} x[n] e^{-j2\pi nk/N_F} \Rightarrow X[2] = \sum_{n=0}^3 x[n] e^{-j\pi n} = a - b + c - d$$

- (b) Find the value of  $X[-7]$  (not  $x[-7]$ ) as a linear combination of  $a, b, c$  and  $d$ .

$$X[-7] = \underline{a - jb - c + jd}$$

$$X[-7] = X[-7+8] = X[1] \Rightarrow X[1] = \sum_{n=0}^3 x[n] e^{-j\pi n/2} = a - jb - c + jd$$

- (c) If the original set of samples was obtained by properly sampling a bandlimited periodic signal above its Nyquist rate over exactly one fundamental period, what is the average value of that signal as a linear combination of  $A, B, C$  and  $D$ ?

$$\text{Average value} = \underline{A/4}$$

$$\text{Average value} = \underline{X[0]/4} = A/4$$

- (d) If  $\{X[0], X[1], X[2], X[3]\} = \{4, 2 - j3, -1, 2 + j3\}$  what is the numerical value of

$$X[-1]? \quad X[-1] = \underline{2 + j3}$$

$$X[-1] = X[-1+4] = X[3] = 2 + j3$$

- (e) If  $X[3] = 1 - j$  what is the numerical value of  $X[-3]$ ?

$$X[-3] = \underline{1 + j}$$

$$X[k] = X^*[-k] \Rightarrow X[-3] = X^*[3] = 1 + j$$