Solution to ECE Test 7 S09

1. With reference to the prototypical feedback system below, find the numerical range of real values of K which make the feedback system stable for the following forward and feedback path transfer functions.

(a) $\mathrm{H}_1(s) = K$, $\mathrm{H}_2(s) = 1/s$

$$\mathrm{H}(s) = \frac{\mathrm{H}_1(s)}{1+\mathrm{H}_1(s)\mathrm{H}_2(s)} = \frac{K}{1+K/s} = \frac{sK}{s+K}$$

Poles at $s+K=0 \Rightarrow s=-K$. We want $\mathrm{Re}(-K)<0$. Therefore for stablity, $K>0$ putting the poles in the left half-plane.

(b) $\mathrm{H}_1(s) = \dfrac{1}{s+3}$, $\mathrm{H}_2(s) = Ks$

$$\mathrm{H}(s) = \frac{\mathrm{H}_1(s)}{1+\mathrm{H}_1(s)\mathrm{H}_2(s)} = \frac{1/(s+3)}{1+Ks/(s+3)} = \frac{1}{s(K+1)+3}$$

Poles at $s(K+1)+3=0 \Rightarrow s=\dfrac{-3}{K+1}$. We want $\mathrm{Re}\left(-\dfrac{3}{K+1}\right)<0 \Rightarrow K+1>0 \Rightarrow K>-1$. Therefore for stablity, $K>-1$ putting the poles in the left half-plane.

(c) $\mathrm{H}_1(s) = \dfrac{K}{s^2+4s+10}$, $\mathrm{H}_2(s) = 1$

$$\mathrm{H}(s) = \frac{\mathrm{H}_1(s)}{1+\mathrm{H}_1(s)\mathrm{H}_2(s)} = \frac{K/(s^2+4s+10)}{1+K/(s^2+4s+10)} = \frac{K}{s^2+4s+10+K}$$

Poles at $s^2+4s+10+K=0 \Rightarrow s=\dfrac{-4\pm\sqrt{16-4(10+K)}}{2}=\dfrac{-4\pm2\sqrt{-6-K}}{2}=-2\pm\sqrt{-6-K}$. We want $\mathrm{Re}\left(-2\pm\sqrt{-6-K}\right)<0$.

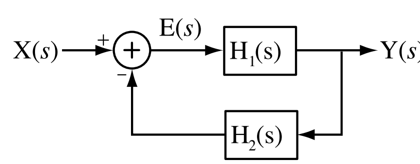
Case 1. $-6-K\ge0 \Rightarrow K\le-6$

The poles are all real and we want $-2\pm\sqrt{-6-K}<0$. If $\sqrt{-6-K}\ge2 \Rightarrow -6-K\ge4 \Rightarrow K\le-10$, then one of the poles will be on the ω axis or in the right half-plane and the system will be unstable. Therefore for stablity, $K>-10$ putting both poles in the left half-plane.

Case 2. $K>-6$

All the poles are complex and the real part is -2 making the system stable.

Overall: For stability, $K>-10$.



Solution to ECE Test 7 S09

1. With reference to the prototypical feedback system below, find the numerical range of real values of K which make the feedback system stable for the following forward and feedback path transfer functions.

(a) $\mathrm{H}_1(s) = K$, $\mathrm{H}_2(s) = 3/s$

$$\mathrm{H}(s) = \frac{\mathrm{H}_1(s)}{1+\mathrm{H}_1(s)\mathrm{H}_2(s)} = \frac{K}{1+3K/s} = \frac{sK}{s+3K}$$

Poles at $s+3K=0 \Rightarrow s=-3K$. We want $\mathrm{Re}(-3K)<0$. Therefore for stablity, $K>0$ putting the poles in the left half-plane.

(b) $\mathrm{H}_1(s) = \dfrac{1}{s+3}$, $\mathrm{H}_2(s) = 2Ks$

$$\mathrm{H}(s) = \frac{\mathrm{H}_1(s)}{1+\mathrm{H}_1(s)\mathrm{H}_2(s)} = \frac{1/(s+3)}{1+2Ks/(s+3)} = \frac{1}{s(2K+1)+3}$$

Poles at $s(2K+1)+3=0 \Rightarrow s=\dfrac{-3}{2K+1}$. We want $\mathrm{Re}\left(-\dfrac{3}{2K+1}\right)<0 \Rightarrow 2K+1>0 \Rightarrow K>-1/2$.

Therefore for stablity, $K>-1/2$ putting the poles in the left half-plane.

(c) $\mathrm{H}_1(s) = \dfrac{K}{s^2+4s+12}$, $\mathrm{H}_2(s) = 1$

$$\mathrm{H}(s) = \frac{\mathrm{H}_1(s)}{1+\mathrm{H}_1(s)\mathrm{H}_2(s)} = \frac{K/(s^2+4s+12)}{1+K/(s^2+4s+12)} = \frac{K}{s^2+4s+12+K}$$

Poles at $s^2+4s+12+K=0 \Rightarrow s=\dfrac{-4\pm\sqrt{16-4(12+K)}}{2}=\dfrac{-4\pm 2\sqrt{-8-K}}{2}=-2\pm\sqrt{-8-K}$. We want $\mathrm{Re}\left(-2\pm\sqrt{-8-K}\right)<0$.

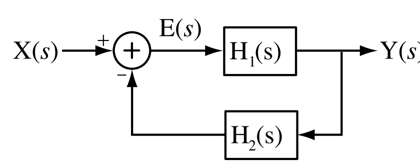
Case 1. $-8-K\geq 0 \Rightarrow K\leq -8$

The poles are all real and we want $-2\pm\sqrt{-8-K}<0$. If $\sqrt{-8-K}\geq 2 \Rightarrow -8-K\geq 4 \Rightarrow K\leq -12$, then one of the poles will be on the ω axis or in the right half-plane and the system will be unstable. Therefore for stablity, $K>-12$ putting both poles in the left half-plane.

Case 2. $K>-8$

All the poles are complex and the real part is -2 making the system stable.

Overall: For stability, $K>-12$.



Solution to ECE Test 7 S09

1. With reference to the prototypical feedback system below, find the numerical range of real values of K which make the feedback system stable for the following forward and feedback path transfer functions.

(a) $\mathrm{H}_1(s) = -K$, $\mathrm{H}_2(s) = 1/s$

$$\mathrm{H}(s) = \frac{\mathrm{H}_1(s)}{1+\mathrm{H}_1(s)\mathrm{H}_2(s)} = \frac{-K}{1-K/s} = \frac{-sK}{s-K}$$

Poles at $s-K=0 \Rightarrow s=K$. We want $\mathrm{Re}(K)<0$. Therefore for stablity, $K<0$ putting the poles in the left half-plane.

(b) $\mathrm{H}_1(s) = \dfrac{2}{s+3}$, $\mathrm{H}_2(s) = 3Ks$

$$\mathrm{H}(s) = \frac{\mathrm{H}_1(s)}{1+\mathrm{H}_1(s)\mathrm{H}_2(s)} = \frac{2/(s+3)}{1+6Ks/(s+3)} = \frac{2}{s(6K+1)+3}$$

Poles at $s(6K+1)+3=0 \Rightarrow s = \dfrac{-3}{6K+1}$. We want $\mathrm{Re}\left(-\dfrac{3}{6K+1}\right)<0 \Rightarrow 6K+1>0 \Rightarrow K>-1/6$.

Therefore for stablity, $K>-1/6$ putting the poles in the left half-plane.

(c) $\mathrm{H}_1(s) = \dfrac{K}{s^2+4s+16}$, $\mathrm{H}_2(s) = 1$

$$\mathrm{H}(s) = \frac{\mathrm{H}_1(s)}{1+\mathrm{H}_1(s)\mathrm{H}_2(s)} = \frac{K/(s^2+4s+16)}{1+K/(s^2+4s+16)} = \frac{K}{s^2+4s+16+K}$$

Poles at $s^2+4s+16+K=0 \Rightarrow s = \dfrac{-4\pm\sqrt{16-4(16+K)}}{2} = \dfrac{-4\pm 2\sqrt{-12-K}}{2} = -2\pm\sqrt{-12-K}$. We want $\mathrm{Re}\left(-2\pm\sqrt{-12-K}\right)<0$.

Case 1. $-12-K \ge 0 \Rightarrow K \le -12$

The poles are all real and we want $-2\pm\sqrt{-12-K}<0$. If $\sqrt{-12-K} \ge 2 \Rightarrow -12-K \ge 4 \Rightarrow K \le -16$, then one of the poles will be on the ω axis or in the right half-plane and the system will be unstable. Therefore for stablity, $K>-16$ putting both poles in the left half-plane.

Case 2. $K>-12$

All the poles are complex and the real part is -2 making the system stable.

Overall: For stability, $K>-16$.