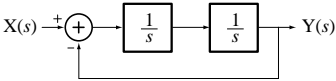


1. Circle the correct answer (marginally-stable is unstable).

(a) Transfer function,  $H(s) = \frac{100s}{s^2 + 3s + 2}$ . Stable

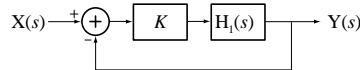
$$H(s) = \frac{100s}{s^2 + 3s + 2} = \frac{100s}{(s+1)(s+2)} \quad \text{Poles in the open left half-plane.}$$

(b)  Unstable

$$s^2 Y(s) = X(s) - Y(s) \Rightarrow Y(s)(s^2 + 1) = X(s) \Rightarrow H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s^2 + 1} = \frac{1}{(s+j)(s-j)}$$

Poles on the  $\omega$  axis. Marginally stable, therefore unstable.

2. For each  $H_1(s)$  is there a finite positive value of  $K$  for which this system is unstable?



(a)  $H_1(s) = 1000 \frac{s-4}{s+10}$

Yes

In the root locus, the closed-loop pole starts on the loop transfer function pole at  $s = -10$  for low  $K$  and as  $K$  increases the closed-loop pole moves toward the loop transfer function zero at  $s = 4$  in the right half-plane. Therefore at some finite positive value of  $K$  the system is unstable.

(b)  $H_1(s) = \frac{25}{s^2 + 2s + 1}$

No

In the root locus, the closed loop poles start at the loop transfer function double pole at  $s = -1$  in the open left-half plane. As  $K$  increases the poles migrate straight up and down and therefore never cross into the right half-plane. Therefore for all  $K$  the system is stable.

(c)  $H_1(s) = -\frac{10}{(s+3)(s+8)(s+22)}$

Yes

There are three real loop transfer function poles in the left half-plane. In the root locus, at some value of  $K$  the closed-loop poles will enter the right half-plane. Therefore at some finite positive value of  $K$  the system is unstable.