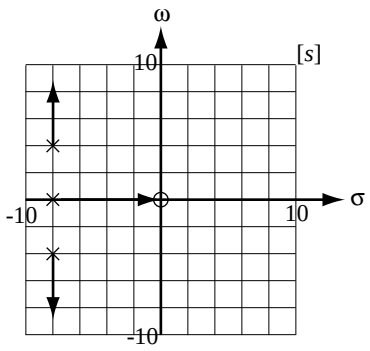
Solution to ECE Test #6 S05

1. If $H_1(s)=\dfrac{s}{s+8}$ and $H_2(s)=\dfrac{1}{s^2+16s+80}$, sketch a root locus in the space provided. (Put a scale on the graph so that actual numerical values can be determined from it.) Is there a finite value of *K* for which this system is unstable? No

Poles at -8 and $-8\pm j4$. One zero at zero.



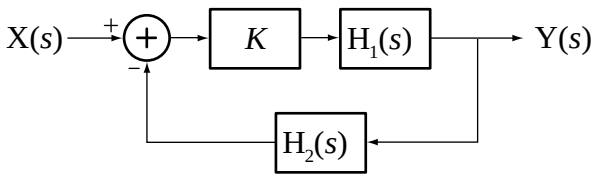
2. If $H_1(s)=\dfrac{1}{s^2-9}$ and $H_2(s)=s^2+4$, for what numerical range of *K* is this system unstable? Range <u>All *K*</u> For what range of *K* is this system marginally stable? Range <u>*K* > 9/4</u>

$$H(s)=\frac{\dfrac{K}{s^2-9}}{1+K\dfrac{s^2+4}{s^2-9}}=\frac{K}{s^2-9+K\left(s^2+4\right)}$$

$$H(s)=\frac{K}{s^2(K+1)+(4K-9)}$$

$$\text{Poles at } s^2(K+1)+(4K-9)=0\Rightarrow s^2=\frac{9-4K}{K+1}\Rightarrow s=\pm\sqrt{\frac{9-4K}{K+1}}$$

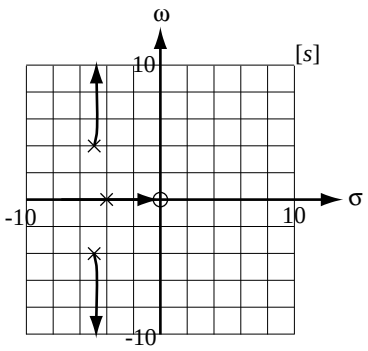
Both poles are real and one is in the right half-plane for any *K* less than 9/4. Poles are imaginary for any *K* greater than 9/4. Therefore the system is unstable for all *K* and is marginally stable for any $K>9/4$.



Solution to ECE Test #6 S05

1. If $H_1(s) = \dfrac{s}{s+4}$ and $H_2(s) = \dfrac{1}{s^2+10s+41}$, sketch a root locus in the space provided. (Put a scale on the graph so that actual numerical values can be determined from it.) Is there a finite value of *K* for which this system is unstable? No

Poles at -4 and $-5 \pm j4$. One zero at zero.



2. If $H_1(s) = \dfrac{1}{s^2-4}$ and $H_2(s) = s^2+9$, for what numerical range of *K* is this system unstable? Range <u>All *K*</u> For what range of *K* is this system marginally stable? Range <u>$K > 4/9$</u>

$$H(s) = \frac{\dfrac{K}{s^2-4}}{1+K\dfrac{s^2+9}{s^2-4}} = \frac{K}{s^2-4+K\left(s^2+9\right)}$$

$$H(s) = \frac{K}{s^2(K+1)+(9K-4)}$$

$$\text{Poles at } s^2(K+1)+(9K-4) = 0 \Rightarrow s^2 = \frac{4-9K}{K+1} \Rightarrow s = \pm\sqrt{\frac{4-9K}{K+1}}$$

Both poles are real and one is in the right half-plane for any *K* less than 4/9. Poles are imaginary for any *K* greater than 4/9. Therefore the system is unstable for all *K* and is marginally stable for any $K > 4/9$.