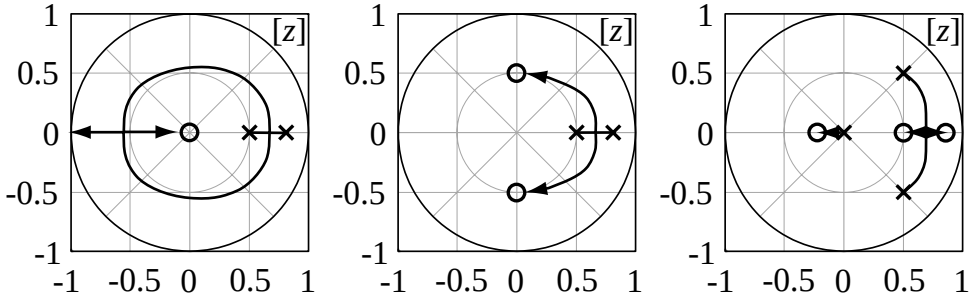
Solution of ECE 316 Test 11 S06

1. Sketch a root locus for each of these pole-zero diagrams of loop transfer functions of feedback systems.



2. A DT system has a transfer function, (the z transform of its impulse response)

$$\mathrm{H}(z)=\frac{0.5z^2}{z^2+1.2z+0.27}\ .$$

If a unit sequence $\mathrm{u}[n]$ is applied as an excitation to this system, what are the numerical values of the responses $\mathrm{y}[0]$, $\mathrm{y}[1]$, and $\mathrm{y}[2]$?

$$\mathrm{Y}(z)=\frac{z}{z-1}\frac{0.5z^2}{z^2+1.2z+0.27}=\frac{0.5z^3}{(z-1)(z+0.9)(z+0.3)}=0.5\left(\frac{0.4049z}{z-1}+\frac{0.7105z}{z+0.9}-\frac{0.1154z}{z+0.3}\right)$$

$$\mathrm{y}[n]=0.5\left(0.4049+0.7105(-0.9)^n-0.1154(-0.3)^n\right)\mathrm{u}[n]$$

$$\mathrm{y}[0]=0.5(0.4049+0.7105-0.1154)=0.5$$
$$\mathrm{y}[1]=0.5\left[0.4049+0.7105(-0.9)-0.1154(-0.3)\right]=-0.1$$
$$\mathrm{y}[2]=0.5\left[0.4049+0.7105(-0.9)^2-0.1154(-0.3)^2\right]=0.485$$

Alternate Solution:

$$\mathrm{Y}(z)=\frac{0.5z^3}{(z-1)(z+0.9)(z+0.3)}=\frac{0.5z^3}{z^3+0.2z^2-0.93z-0.27}$$

$$
\begin{array}{r}
0.5 - 0.1z^{-1} + 0.485z^{-2} \\
z^3 + 0.2z^2 - 0.93z - 0.27 \overline{)\,0.5z^3} \\
\underline{0.5z^3 + 0.1z^2 - 0.465z - 0.135} \\
-0.1z^2 + 0.465z + 0.135 \\
\underline{-0.1z^2 - 0.02z + 0.093 + 0.027z^{-1}} \\
0.485z + 0.38 - 0.027z^{-1}
\end{array}
$$

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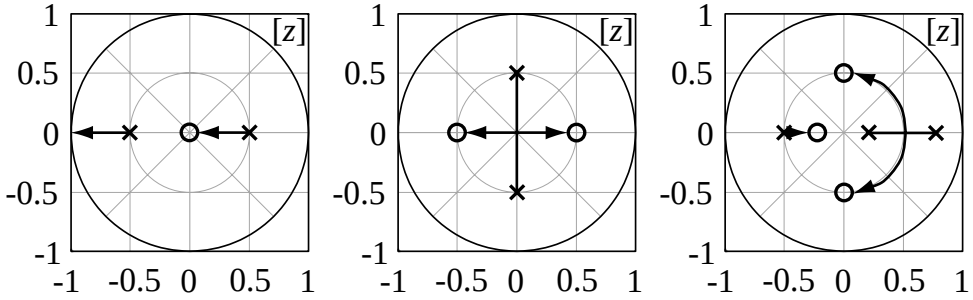
1. Sketch a root locus for each of these pole-zero diagrams of loop transfer functions of feedback systems.



2. A DT system has a transfer function, (the z transform of its impulse response)

$$\mathrm{H}(z)=\frac{0.4z^2}{z^2+1.3z+0.4}\ .$$

If a unit sequence $\mathrm{u}[n]$ is applied as an excitation to this system, what are the numerical values of the responses $\mathrm{y}[0]$, $\mathrm{y}[1]$, and $\mathrm{y}[2]$?

$$\mathrm{Y}(z)=\frac{z}{z-1}\frac{0.4z^2}{z^2+1.3z+0.4}=\frac{0.4z^3}{(z-1)(z+0.8)(z+0.5)}=0.4\left(\frac{0.1481z}{z-1}+\frac{0.4741z}{z+0.8}-\frac{0.2222z}{z+0.5}\right)$$

$$\mathrm{y}[n]=0.4\left(0.1481+0.4741(-0.8)^n-0.2222(-0.5)^n\right)\mathrm{u}[n]$$

$$\mathrm{y}[0]=0.4(0.1481+0.4741-0.2222)=0.4$$
$$\mathrm{y}[1]=0.4\left[0.1481+0.4741(-0.8)-0.2222(-0.5)\right]=-0.12$$
$$\mathrm{y}[2]=0.4\left[0.1481+0.4741(-0.8)^2-0.2222(-0.5)^2\right]=0.396$$

Alternate Solution:

$$\mathrm{Y}(z)=\frac{z}{z-1}\frac{0.4z^2}{z^2+1.3z+0.4}=\frac{0.4z^3}{z^3+0.3z^2-0.9z-0.4}$$

$$
\begin{array}{r}
0.4 - 0.12z^{-1} + 0.396z^{-2} \\
z^3 + 0.3z^2 - 0.9z - 0.4 \overline{)\, 0.4z^3} \\
\underline{0.4z^3 + 0.12z^2 - 0.36z - 0.16} \\
-0.12z^2 + 0.36z + 0.16 \\
\underline{-0.12z^2 - 0.036z + 0.108 + 0.048z^{-1}} \\
0.396z + 0.052 - 0.048z^{-1}
\end{array}
$$