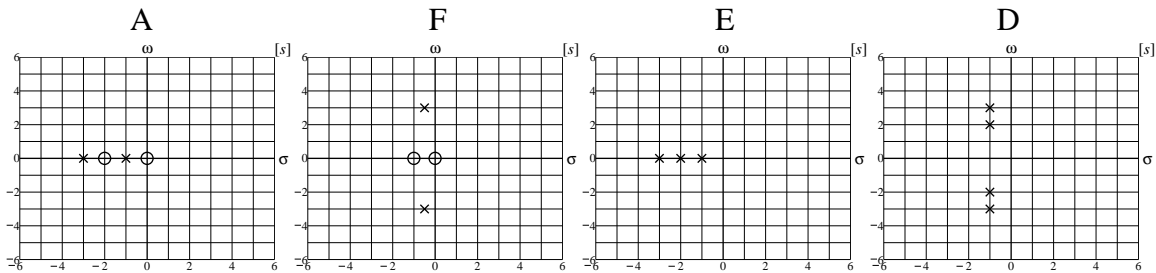# Solution to ECE Test #3 Su06

1. For each pole-zero diagram choose the corresponding frequency response and write its letter designation in the space provided. (In each case assume that the transfer function is of the form $H(s) = K \dfrac{s^N + b_{N-1}s^{N-1} + \quad + b_1 s + b_0}{s^D + a_{D-1}s^{D-1} + \quad + a_1 s + a_0}$ and that $K = 1$.)


A   F   E   D

The first two are both basically highpass filters but in the second one the two poles are very near the ω axis causing the magnitude response to peak sharply there as in F. Therefore the first one is A and the second one is F.

The last two are both basically lowpass filters, one with three finite poles and the other with four finite poles. Also the poles in the last one are complex and relatively close to the ω axis causing some peaking again between 2 and 3 in ω. So the third one is E and the last one is D. The effect of the number of finite poles can also be seen in the phase graphs for E and D.
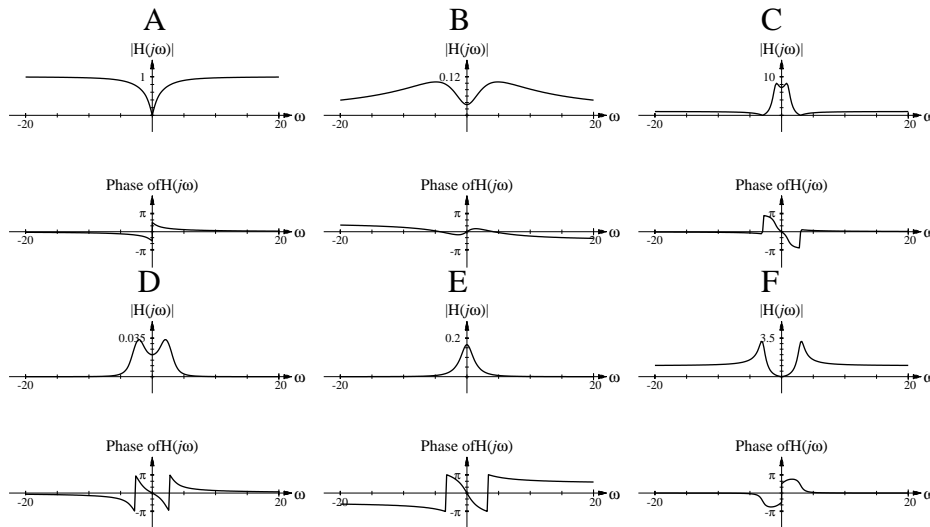
2. These pole-zero diagrams are for the loop transfer function $T(s)$ of a feedback system.

    (a) Sketch a root-locus for each one.

A

B

C

D
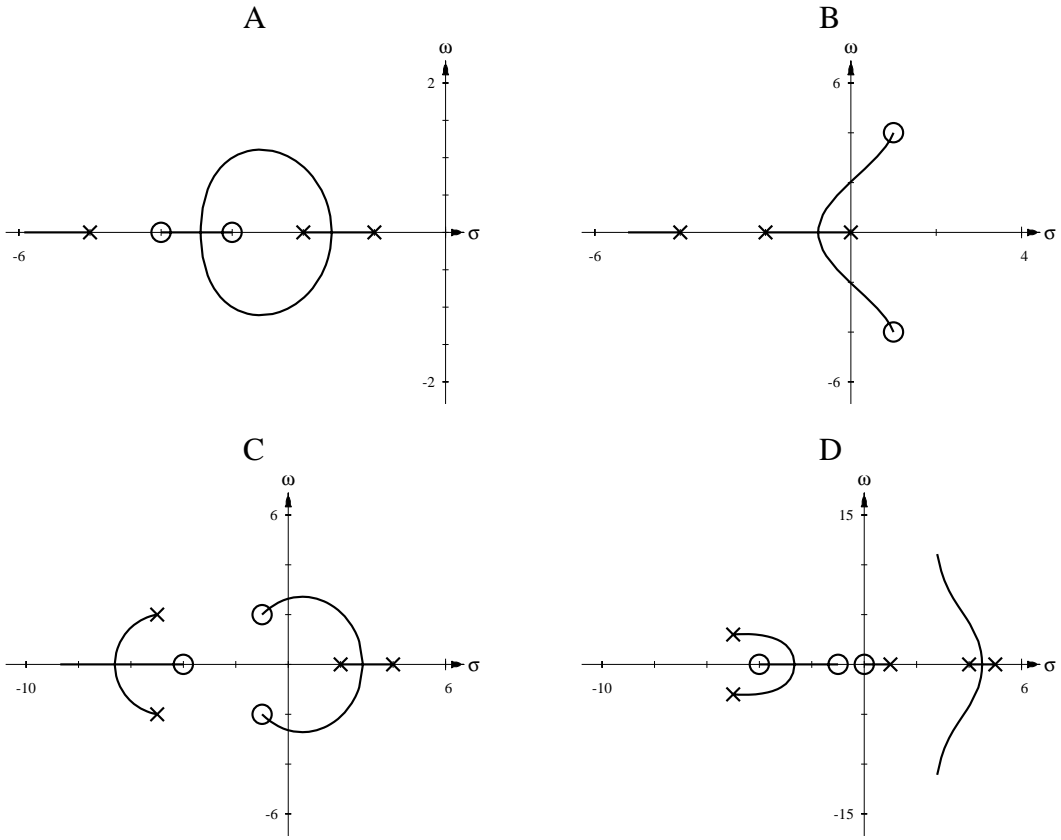
    (b) Choose whether or not each system can go unstable at a finite positive real value of the gain constant K.

    A   Stable for all finite positive real K __X__
         Not stable for all finite positive real K _____

    B   Stable for all finite positive real K _____
         Not stable for all finite positive real K __X__

    C   Stable for all finite positive real K _____
         Not stable for all finite positive real K __X__

    D   Stable for all finite positive real K _____
         Not stable for all finite positive real K __X__

    (c) Which of these systems are unstable for small positive real values of K but become stable for larger positive real values of K?   __C__

3.   (A unity-gain feedback system has a forward-path transfer function

$$H_1(s) = \frac{K}{s^n(s+7)}.$$

$$H(s) = \frac{\dfrac{K}{s^n(s+7)}}{1 + \dfrac{K}{s^n(s+7)}} = \frac{K}{s^n(s+7)+K}$$

(a)   If $n = 0$, what range of positive real K's makes the overall feedback system stable?

$$H(s) = \frac{K}{s+7+K} \Rightarrow 0 < K < \infty \text{ for stability}$$

(b)   If $n = 1$ what range of positive real K's makes the overall feedback system have real poles?

$$H(s) = \frac{K}{s(s+7)+K} = \frac{K}{s^2+7s+K} \Rightarrow \text{Poles at } \frac{-7 \pm \sqrt{49-4K}}{2} \Rightarrow 0 < K < 49/4$$

(c)   If $n = 0$, $K = 10$ and the system is excited by a unit step, what is the numerical steady-state error?

$$H_{-1}(s) = \frac{10}{s(s+7+10)} = \frac{10}{s(s+17)} = \frac{10/17}{s} - \frac{10/17}{s+17} \Rightarrow h_{-1}(t) = (10/17)(1 - e^{-17t})u(t)$$

The final value of the step response is 10/17. Therefore the steady-state error is $1 - 10/17 = 7/17$.

(d)   If $n = 1$, $K = 10$ and the system is excited by a unit step, what is the numerical steady-state error?

Type 1 system $\Rightarrow$ Steady-state error is 0.

4.   A system has a transfer function $H(s) = \dfrac{s(s^2-4)}{(s+4)(s+8)(s+14)}$.

(a)   Draw a system block diagram realization for this system in the canonical form.

$$H(s) = \frac{s(s^2-4)}{(s+4)(s+8)(s+14)} = \frac{s^3-4s}{s^3+26s^2+200s+448}$$
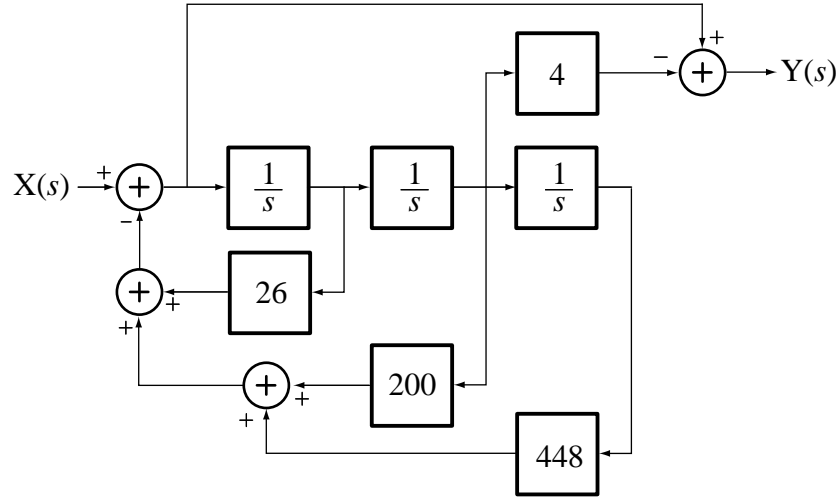
(b)   Draw a system block diagram realization for this system in the cascade form.

$$H(s) = \frac{s(s^2-4)}{(s+4)(s+8)(s+14)} = \frac{s}{s+4} \times \frac{s+2}{s+8} \times \frac{s-2}{s+14}$$