Solution of ECE 316 Test 8 S06

1. What are the numerical s-plane finite pole and finite zero locations for Butterworth filters of order N with corner frequencies f_c in Hz or ω_c in radians/second. (For repeated poles or zeros list them multiple times.)

(a) Lowpass,
$$N = 2$$
, $\omega_c = 25$
Poles at $25e^{\pm j3\pi/4} = 17.68(-1 \pm j)$ and No Zeros
$$H_{norm}(s) = \frac{1}{\left(s - e^{j3\pi/4}\right)\left(s - e^{-j3\pi/4}\right)} \xrightarrow{s \to s/25} H(s) = \frac{1}{\left(s/25 - e^{j3\pi/4}\right)\left(s/25 - e^{-j3\pi/4}\right)}$$

$$H(s) = \frac{625}{\left(s - 25e^{j3\pi/4}\right)\left(s - 25e^{-j3\pi/4}\right)}$$
, Poles at $25e^{\pm j3\pi/4} = 17.68(-1 \pm j)$

No finite zeros.

(b) Highpass, N=2, $f_c=5$ Poles at $10\pi/e^{\pm j3\pi/4}=22.2(1\pm j)$ and Double Zero at zero.

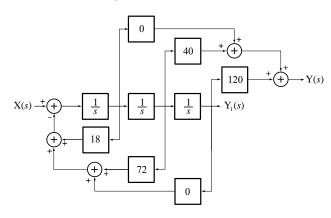
$$H_{norm}(s) = \frac{1}{\left(s - e^{j3\pi/4}\right)\left(s - e^{-j3\pi/4}\right)} \xrightarrow{s \to 10\pi/s} H(s) = \frac{1}{\left(10\pi/s - e^{j3\pi/4}\right)\left(10\pi/s - e^{-j3\pi/4}\right)}$$

$$H(s) = \frac{s^2/\left(10\pi\right)^2}{\left(1 - se^{j3\pi/4}/10\pi\right)\left(1 - se^{-j3\pi/4}/10\pi\right)} , \text{ Poles at } 10\pi/e^{\pm j3\pi/4} = 22.2\left(1 \pm j\right)$$

2. Put numbers in the standard third-order CT system canonical block diagram below for the system whose transfer function is

$$H(s) = \frac{40(s+3)}{s(s+6)(s+12)} = \frac{40s+120}{s^3+18s^2+72s}.$$

(Put a number in each gain block even if it is zero.)



Solution of ECE 316 Test 8 S06

1. What are the numerical s-plane finite pole and finite zero locations for Butterworth filters of order N with corner frequencies f_c in Hz or ω_c in radians/second. (For repeated poles or zeros list them multiple times.)

(a) Lowpass,
$$N=2$$
, $f_c=25$ $\omega_c=2\pi f_c=50\pi$
Poles at $50\pi e^{\pm j3\pi/4}=111.07(-1\pm j)$ and No Zeros

$$H_{norm}(s) = \frac{1}{\left(s - e^{j3\pi/4}\right)\left(s - e^{-j3\pi/4}\right)} \xrightarrow{s \to s/50\pi} H(s) = \frac{1}{\left(s/50\pi - e^{j3\pi/4}\right)\left(s/50\pi - e^{-j3\pi/4}\right)}$$

$$H(s) = \frac{\left(50\pi\right)^2}{\left(s - 50\pi e^{j3\pi/4}\right)\left(s - 50\pi e^{-j3\pi/4}\right)} , \text{ Poles at } 50\pi e^{\pm j3\pi/4} = 111.07\left(-1 \pm j\right)$$

No finite zeros.

(b) Highpass, N=2, $\omega_c=5$ Poles at $5/e^{\pm j3\pi/4}=3.53(1\pm j)$ and Double Zero at zero.

$$H_{norm}(s) = \frac{1}{(s - e^{j3\pi/4})(s - e^{-j3\pi/4})} \xrightarrow{s \to 5/s} H(s) = \frac{1}{(5/s - e^{j3\pi/4})(5/s - e^{-j3\pi/4})}$$

$$H(s) = \frac{s^2 / 25}{(1 - se^{j3\pi/4} / 5)(1 - se^{-j3\pi/4} / 5)}, \text{ Poles at } 5/e^{\pm j3\pi/4} = 3.53(1 \pm j)$$

2. Put numbers in the standard third-order CT system canonical block diagram below for the system whose transfer function is

$$H(s) = \frac{20(s+2)}{s(s+5)(s+11)} = \frac{20s+40}{s^3+16s^2+55s}.$$

(Put a number in each gain block even if it is zero.)

