Solution to ECE Test #12 S07 #1

A continuous-time filter has a transfer function $\mathrm{H}(s) = 4\dfrac{s-2}{s(s+2)}$. It is approximated by three digital filter design methods, matched z transform, direct substitution and bilinear z transform using a sampling rate $f_s = 2$. What are the numerical pole and zero locations of these digital filters?

Matched z Transform — zeros at <u>0, 2.718</u> , poles at <u>1, 0.368</u>

Direct Substitution — zeros at <u>2.718</u> , poles at <u>1, 0.368</u>

Bilinear z Transform — zeros at <u>3, -1</u> , poles at <u>1, 0.333</u>

Matched z Transform

$$\mathrm{H}(z) = 4\frac{1-e^{1}z^{-1}}{\left(1-z^{-1}\right)\left(1-e^{-1}z^{-1}\right)} = 4\frac{z(z-2.718)}{(z-1)(z-0.368)}$$

Direct Substitution

$$\mathrm{H}(z) = 4\frac{z-e^{1}}{(z-1)\left(z-e^{-1}\right)} = 4\frac{z-2.718}{(z-1)(z-0.368)}$$

Bilinear

$$\mathrm{H}(z) = 4\frac{4\frac{z-1}{z+1}-2}{4\frac{z-1}{z+1}\left(4\frac{z-1}{z+1}+2\right)} = \frac{4z-4-2z-2}{\frac{z-1}{z+1}(4z-4+2z+2)}$$

$$\mathrm{H}(z) = \frac{2z-6}{\frac{z-1}{z+1}(6z-2)} = \frac{(2z-6)(z+1)}{(z-1)(6z-2)} = \frac{1}{3}\frac{(z-3)(z+1)}{(z-1)(z-1/3)}$$

Solution to ECE Test #12 S07 #2

A continuous-time filter has a transfer function $\mathrm{H}(s) = 4\frac{s-1}{s(s+1)}$. It is approximated by three digital filter design methods, matched z transform, direct substitution and bilinear z transform using a sampling rate $f_s = 0.8$. What are the numerical pole and zero locations of these digital filters?

Matched z Transform zeros at <u>0, 3.49</u> , poles at <u>1, 0.2865</u>

Direct Substitution zeros at <u>3.49</u> , poles at <u>1, 0.2865</u>

Bilinear z Transform zeros at <u>4.333, -1</u> , poles at <u>1, 0.2308</u>

Matched z Transform

$$\mathrm{H}(z) = 4\frac{1 - e^{1.25}z^{-1}}{\left(1 - z^{-1}\right)\left(1 - e^{-1.25}z^{-1}\right)} = 4\frac{z(z - 3.49)}{(z - 1)(z - 0.2865)}$$

Direct Substitution

$$\mathrm{H}(z) = 4\frac{z - e^{1.25}}{(z - 1)\left(z - e^{-1.25}\right)} = 4\frac{z - 3.49}{(z - 1)(z - 0.2865)}$$

Bilinear

$$\mathrm{H}(z) = 4\frac{1.6\frac{z-1}{z+1} - 1}{1.6\frac{z-1}{z+1}\left(1.6\frac{z-1}{z+1} + 1\right)} = \frac{4}{1.6}\frac{1.6z - 1.6 - z - 1}{\frac{z-1}{z+1}(1.6z - 1.6 + z + 1)}$$

$$\mathrm{H}(z) = 2.5\frac{0.6z - 2.6}{\frac{z-1}{z+1}(2.6z - 0.6)} = 2.5\frac{(0.6z - 2.6)(z + 1)}{(z - 1)(2.6z - 0.6)} = 0.577\frac{(z - 4.333)(z + 1)}{(z - 1)(z - 0.2308)}$$

Solution to ECE Test #12 S07 #3

A continuous-time filter has a transfer function $\mathrm{H}(s) = 4\frac{s-3}{s(s+3)}$. It is approximated by three digital filter design methods, matched z transform, direct substitution and bilinear z transform using a sampling rate $f_s = 2.5$. What are the numerical pole and zero locations of these digital filters?

Matched z Transform zeros at <u>0, 3.32</u> , poles at <u>1, 0.301</u>

Direct Substitution zeros at <u>3.32</u> , poles at <u>1, 0.301</u>

Bilinear z Transform zeros at <u>4, -1</u> , poles at <u>1, 0.25</u>

Matched z Transform

$$\mathrm{H}(z) = 4\frac{1-e^{1.2}z^{-1}}{(1-z^{-1})(1-e^{-1.2}z^{-1})} = 4\frac{z(z-3.32)}{(z-1)(z-0.301)}$$

Direct Substitution

$$\mathrm{H}(z) = 4\frac{z-e^{1.2}}{(z-1)(z-e^{-1.2})} = 4\frac{z-3.32}{(z-1)(z-0.301)}$$

Bilinear

$$\mathrm{H}(z) = 4\frac{5\frac{z-1}{z+1}-3}{5\frac{z-1}{z+1}\left(5\frac{z-1}{z+1}+3\right)} = 0.8\frac{5z-5-3z-3}{\frac{z-1}{z+1}(5z-5+3z+3)}$$

$$\mathrm{H}(z) = 0.8\frac{2z-8}{\frac{z-1}{z+1}(8z-2)} = 0.8\frac{(2z-8)(z+1)}{(z-1)(8z-2)} = 0.2\frac{(z-4)(z+1)}{(z-1)(z-1/4)}$$