Solution to ECE Test #12 S08 #1

A digital filter is designed using the direct substitution method ($s-a \Rightarrow z-e^{aT_s}$) to approximate an analog filter with poles at $s=0$ and at $s=-3\pm j2$ and no finite zeros.

(a) If the sampling rate is $f_s = 20$, where are the poles of the digital filter in the z plane?

$$\mathrm{H}(s) = \frac{K}{s(s+3-j2)(s+3+j2)} = \frac{K}{s\left(s^2+6s+13\right)}$$

$$T_s = 1/20$$

$$s-0 \Rightarrow z-e^{j0} = z-1$$

$$s+3\pm j2 \Rightarrow z-e^{(-3\pm j2)/20} = z-0.8564 \pm j0.0859 = z-0.8607e^{\pm j0.1}$$

$$\mathrm{H}(z) = \frac{K}{(z-1)(z-0.8654+j0.0859)(z-0.8654-j0.0859)} = \frac{K}{(z-1)\left(z^2-1.7128s+0.7408\right)}$$

Poles at $z=1$ and $z = 0.8564 \pm j0.0859 = 0.8607e^{\pm j0.1}$

(b) If the method is changed to the matched filter method ($s-a \Rightarrow 1-e^{aT_s}z^{-1}$), where are the zeros of the digital filter in the z plane?

$$s-0 \Rightarrow 1-z^{-1} = \frac{z-1}{z} \quad , \quad s+3\pm j2 \Rightarrow 1-(0.8564 \pm j0.0859)z^{-1} = \frac{z-(0.8564 \pm j0.0859)}{z}$$

$$\mathrm{H}(z) = \frac{Kz^3}{(z-1)(z-0.8654+j0.0859)(z-0.8654-j0.0859)} = \frac{Kz^3}{(z-1)\left(z^2-1.7128s+0.7408\right)}$$

Three finite zeros at $z=0$.

Solution to ECE Test #12 S08 #2

A digital filter is designed using the direct substitution method ($s-a \Rightarrow z-e^{aT_s}$) to approximate an analog filter with poles at $s=0$ and at $s=-3\pm j2$ and no finite zeros.

(a) If the sampling rate is $f_s=10$, where are the poles of the digital filter in the z plane?

$$\mathrm{H}(s)=\frac{K}{s(s+3-j2)(s+3+j2)}=\frac{K}{s\left(s^2+6s+13\right)}$$

$$T_s=1/10$$

$$s-0 \Rightarrow z-e^{j0}=z-1$$

$$s+3\pm j2 \Rightarrow z-e^{(-3\pm j2)/10}=z-0.7261\pm j0.1472=z-0.7408e^{\pm j0.2}$$

$$\mathrm{H}(z)=\frac{K}{(z-1)(z-0.7261+j0.1472)(z-0.7261-j0.1472)}=\frac{K}{(z-1)\left(z^2-1.4522s+0.5488\right)}$$

Poles at $z=1$ and $z=0.7261\pm j0.1472=0.7408e^{\pm j0.2}$

(b) If the method is changed to the matched filter method ($s-a \Rightarrow 1-e^{aT_s}z^{-1}$), where are the zeros of the digital filter in the z plane?

$$s-0 \Rightarrow 1-z^{-1}=\frac{z-1}{z}\ ,\ s+3\pm j2 \Rightarrow 1-(0.7261\pm j0.1472)z^{-1}=\frac{z-(0.7261\pm j0.1472)}{z}$$

$$\mathrm{H}(z)=\frac{Kz^3}{(z-1)(z-0.7261+j0.1472)(z-0.7261-j0.1472)}=\frac{Kz^3}{(z-1)\left(z^2-1.4522s+0.5488\right)}$$

Three finite zeros at $z=0$.

Solution to ECE Test #12 S08 #3

A digital filter is designed using the direct substitution method ($s - a \Rightarrow z - e^{aT_s}$) to approximate an analog filter with poles at $s = 0$ and at $s = -2 \pm j3$ and no finite zeros.

(a) If the sampling rate is $f_s = 20$, where are the poles of the digital filter in the z plane?

$$\mathrm{H}(s) = \frac{K}{s(s+2-j3)(s+2+j3)} = \frac{K}{s(s^2+4s+13)}$$

$$T_s = 1/20$$

$$s - 0 \Rightarrow z - e^{j0} = z - 1$$

$$s + 2 \pm j3 \Rightarrow z - e^{(-2 \pm j3)/20} = z - 0.8947 \pm j0.1352 = z - 0.9048e^{\pm j0.15}$$

$$\mathrm{H}(z) = \frac{K}{(z-1)(z-0.8947+j0.1352)(z-0.8947-j0.1352)} = \frac{K}{(z-1)(z^2-1.7894s+0.8187)}$$

Poles at $z = 1$ and $z = 0.8947 \pm j0.1352 = 0.9048e^{\pm j0.15}$

(b) If the method is changed to the matched filter method ($s - a \Rightarrow 1 - e^{aT_s}z^{-1}$), where are the zeros of the digital filter in the z plane?

$$s - 0 \Rightarrow 1 - z^{-1} = \frac{z\text{ - }1}{z} \;\;,\;\; s + 2 \pm j3 \Rightarrow 1 - (0.8947 \pm j0.1352)z^{-1} = \frac{z - (0.8947 \pm j0.1352)}{z}$$

$$\mathrm{H}(z) = \frac{Kz^3}{(z-1)(z-0.8947+j0.1352)(z-0.8947-j0.1352)} = \frac{Kz^3}{(z-1)(z^2-1.7894s+0.8187)}$$

Three finite zeros at $z = 0$.