Solution of ECE 316 Test 12 S06

A CT system has a transfer function $\mathrm{H}(s)=\dfrac{20s}{s^2+7s+12}$.

(a) Its impulse response can be expressed in the form $\mathrm{h}(t)=\left(Ae^{at}+Be^{bt}\right)\mathrm{u}(t)$. Find the numerical values of A, B, a and b.

$$A = 80 \ , \ B = \text{-}60 \ , \ a = \text{-}4 \ , \ b = \text{-}3$$

$$\mathrm{H}(s)=\frac{20s}{s^2+7s+12}=20\frac{s}{(s+4)(s+3)}=20\left[\frac{4}{s+4}-\frac{3}{s+3}\right]\Rightarrow \mathrm{h}(t)=20\left(4e^{-4t}-3e^{-3t}\right)\mathrm{u}(t)$$

(b) Using the impulse invariant technique with a sampling rate of $f_s = 10$ Hz the impulse response of the corresponding DT system can be expressed in the form $\mathrm{h}[n]=\left(Aa^n+Bb^n\right)\mathrm{u}[n]$. Find the numerical values of A, B, a and b.

$$A = 80 \ , \ B = \text{-}60 \ , \ a = 0.6703 \ , \ b = 0.7408$$

$$\begin{aligned}\mathrm{h}[n] &= 20\left(4e^{-4nT_s}-3e^{-3nT_s}\right)\mathrm{u}[nT_s]=20\left(4e^{-0.4n}-3e^{-0.3n}\right)\mathrm{u}[n]\\ &= 20\left[4\left(0.6703\right)^n-3\left(0.7408\right)^n\right]\mathrm{u}[n]\end{aligned}$$

(c) The z-domain transfer function can be expressed in the form $\mathrm{H}(z)=z\dfrac{Az+B}{z^2+az+b}$. Find the numerical values of A, B, a and b

$$A = 20 \ , \ B = \text{-}19.046, \ a = \text{-}1.411 \ , \ b = 0.4966$$

$$\mathrm{H}(z)=20\left[\frac{4z}{z-0.6703}-\frac{3z}{z-0.7408}\right]=20\frac{z(z-0.9523)}{z^2-1.411z+0.4966}$$

Solution of ECE 316 Test 12 S06

A CT system has a transfer function $\mathrm{H}(s)=\dfrac{4s}{s^2+11s+30}$.

(a) Its impulse response can be expressed in the form $\mathrm{h}(t)=\left(Ae^{at}+Be^{bt}\right)\mathrm{u}(t)$. Find the numerical values of A, B, a and b.

$A = 24$, $B = -20$, $a = -6$, $b = -5$

$$\mathrm{H}(s)=\frac{4s}{s^2+11s+30}=4\frac{s}{(s+6)(s+5)}=4\left[\frac{6}{s+6}-\frac{5}{s+5}\right]\Rightarrow \mathrm{h}(t)=4\left(6e^{-6t}-5e^{-5t}\right)\mathrm{u}(t)$$

(b) Using the impulse invariant technique with a sampling rate of $f_s = 10$ Hz the impulse response of the corresponding DT system can be expressed in the form $\mathrm{h}[n]=\left(Aa^n+Bb^n\right)\mathrm{u}[n]$. Find the numerical values of A, B, a and b.

$A = 24$, $B = -20$, $a = 0.5488$, $b = 0.6065$

$$\begin{aligned}\mathrm{h}[n] &= 4\left(6e^{-6nT_s}-5e^{-5nT_s}\right)\mathrm{u}[nT_s]=4\left(6e^{-0.6n}-5e^{-0.5n}\right)\mathrm{u}[n]\\ &= 4\left[6(0.5488)^n-5(0.6065)^n\right]\mathrm{u}[n]\end{aligned}$$

(c) The z-domain transfer function can be expressed in the form $\mathrm{H}(z)=z\dfrac{Az+B}{z^2+az+b}$. Find the numerical values of A, B, a and b

$A = 4$, $B = -3.58$, $a = -1.1553$, $b = 0.3328$

$$\mathrm{H}(z)=4\left[\frac{6z}{z-0.5488}-\frac{5z}{z-0.6065}\right]=4\frac{z(z-0.8950)}{z^2-1.1553z+0.3328}$$