

Solution of ECE 316 Test #12 S04 # 1

1. An FIR filter has an impulse response, $h[n] = \frac{\delta[n] + \delta[n-1] + \delta[n-2]}{10}$.

(a) (6 pts) How many finite poles and finite zeros are there in its transfer function and what are their numerical locations?

$$H(z) = 0.1[1 + z^{-1} + z^{-2}] = 0.1\left[1 + \frac{1}{z} + \frac{1}{z^2}\right] = 0.1 \frac{z^2 + z + 1}{z^2} = 0.1 \frac{\left(z - \frac{1}{2} + j\frac{\sqrt{3}}{2}\right)\left(z - \frac{1}{2} - j\frac{\sqrt{3}}{2}\right)}{z^2}$$

Two zeros at $z = \frac{1}{2} \pm \frac{\sqrt{3}}{2}$ and a two poles at $z = 0$.

(b) (4 pts) If the excitation, $x[n]$, of this system is a unit sequence, what is the final numerical value of the response, $\lim_{n \rightarrow \infty} y[n]$? 0.3

$$\lim_{n \rightarrow \infty} y[n] = \lim_{z \rightarrow 1} (z-1) \frac{z}{z-1} H(z) = 0.1 \lim_{z \rightarrow 1} \frac{z^2 + z + 1}{z} = 0.3$$

2. (5 pts) An FIR filter has an impulse response, $h[n] = \frac{\delta[n] + \delta[n-1]}{2}$. A DT sinusoid, $x[n]$, is created by sampling, at $f_s = 10\text{Hz}$, a CT sinusoid with cyclic frequency, f_0 . What is the minimum positive numerical value of f_0 for which the filter response is zero? 5 Hz

$$H(z) = 0.5\left[1 + \frac{1}{z}\right] = 0.5 \frac{z+1}{z} \Rightarrow H(e^{j2\pi f_0 T_s}) = 0.5 \frac{e^{j2\pi f_0 T_s} + 1}{e^{j2\pi f_0 T_s}} = 0.5 \frac{e^{j0.2\pi f_0} + 1}{e^{j0.2\pi f_0}} = 0$$

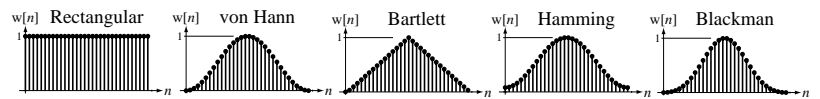
$$e^{j0.2\pi f_0} + 1 = 0 \Rightarrow 0.2\pi f_0 = \pi \Rightarrow f_0 = 5$$

Alternate Solution: If the impulse response is $h[n] = \frac{\delta[n] + \delta[n-1]}{2}$, that is simply saying that the response to any signal is always the average of the present value (at time, n) and the previous value (at time, $n-1$). First, think about a very low frequency sinusoid, say 1 Hz, sampled at 10 Hz. The time between samples is then 1/10 of a period. These two values might cancel occasionally but not every time, so the response would not be zero. But as you increase the frequency, you eventually reach a frequency at which the time between samples is exactly half a period. At that frequency, the present sample and the previous sample are always negatives of each other. If the time between samples is half a period that means a full period is twice the time between samples or 0.2 seconds. That period corresponds to a frequency of 5 Hz.

3. (1 pt) Below are the window shapes of five common digital filter windows. If these windows are used to truncate an ideal lowpass filter impulse response, which one will have the greatest stopband attenuation?

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(Let the stop band be defined as the range of frequencies greater than the first null in the frequency response.)



Look at the transforms of these window shapes in the book.