Solution of ECE 316 Test #11 S03 4/9/03 #1

1. A digital filter designed by the impulse invariant design method has an impulse response, $\mathrm{h}[n] = 0.8^n \mathrm{u}[n]$. It simulates a CT filter with an impulse response of $\mathrm{h}(t) = e^{-4t}\mathrm{u}(t)$. What is the sampling rate, f_s?

For the impulse invariant method,
$0.8^n \mathrm{u}[n] = e^{-4nT_s}\mathrm{u}(nT_s) \Rightarrow 0.8 = e^{-4T_s} \Rightarrow T_s = 0.05578\ \mathrm{s} \Rightarrow f_s = 17.93\ \mathrm{Hz}$

2. A digital filter has an impulse response, $\mathrm{h}[n] = 0.6^n \mathrm{u}[n]$. If it is excited by a unit sequence, what is the final value of the response? $\left(\lim_{n\to\infty} \mathrm{g}[n] = \lim_{z\to 1}(z-1)\mathrm{G}(z)\right)$

$\mathrm{H}(z) = \frac{z}{z-0.6} \Rightarrow \mathrm{H}_{-1}(z) = \frac{z}{z-1}\frac{z}{z-0.6} \Rightarrow \lim_{n\to\infty}\mathrm{h}_{-1}[n] = \lim_{z\to 1}(z-1)\frac{z}{z-1}\frac{z}{z-0.6} = \frac{1}{1-0.6} = 2.5$
($\mathrm{h}_{-1}[n]$ is the unit sequence response)

3. A digital filter has a transfer function, $\mathrm{H}(z) = \frac{10z}{z-0.5}$. At what DT radian frequency, Ω, is its magnitude response a minimum? $\left(z = e^{j\Omega}\right)$

$\mathrm{H}(e^{j\Omega}) = \frac{10e^{j\Omega}}{e^{j\Omega}-0.5}$. Minimum magnitude response occurs where $e^{j\Omega} - 0.5$ is a maximum magnitude which is at $\Omega = \pm\pi$. (Accept either $\Omega = \pi$ or $\Omega = -\pi$ as correct.)

4. A digital filter has a transfer function, $\mathrm{H}(z) = \frac{10(z-1)}{z-0.3}$. At what DT radian frequency, Ω, is its magnitude response a minimum? $\left(z = e^{j\Omega}\right)$

$\mathrm{H}(e^{j\Omega}) = \frac{10\left(e^{j\Omega}-1\right)}{e^{j\Omega}-0.3}$. Minimum magnitude occurs where $e^{j\Omega} - 1 = 0$ which is at $\Omega = 0$.

5. A digital filter has a transfer function, $\mathrm{H}(z) = \frac{2z}{z-0.7}$. What is the magnitude of its response at a DT radian frequency of $\Omega = \frac{\pi}{2}$? $\left(z = e^{j\Omega}\right)$

$$\mathrm{H}(e^{j\Omega}) = \frac{2e^{j\Omega}}{e^{j\Omega}-0.7} = \frac{2e^{j\frac{\pi}{2}}}{e^{j\frac{\pi}{2}}-0.7} = \frac{j2}{j-0.7} = 1.342 - j0.9396 \Rightarrow \left|\mathrm{H}\left(e^{j\frac{\pi}{2}}\right)\right| = 1.6385$$