Solution to ECE Test #12 S09

1. The device which accepts an impulse excitation, produces a rectangular response and is used in modeling the action of a DAC is called a <u>zero-order hold.</u>
2. Digital filters whose impulse response goes to zero at a finite time and stays there forever after that are called <u>finite-duration impulse response (FIR) filters</u>
3. Digital filters whose impulse response approaches zero asymptotically as time approaches infinity are called <u>infinite-duration impulse response (IIR) filters</u>

4. When trying to design an approximation to an ideal differentiator ($\mathrm{y}(t) = \frac{d}{dt}(\mathrm{x}(t))$) using the step invariant technique we encounter a major problem.

$$\mathrm{Y}(s) = s\,\mathrm{X}(s) \Rightarrow \mathrm{H}(s) = s \Rightarrow \mathrm{H}_{-1}(s) = s \times 1/s = 1 \Rightarrow \mathrm{h}_{-1}(t) = \delta(t)$$

The next step would be to sample the CT step response. But, since the step response is a CT impulse it cannot be sampled.

5. A lowpass analog filter has unity gain at a frequency of zero and a single real pole at $s = -20$. A digital filter is designed to approximate it using the impulse-invariant technique with a sampling rate of 200 Hz. Find the following.

$\mathrm{h}(t) = 20e^{-20t}\,\mathrm{u}(t)$ $\qquad$ $\mathrm{h}[n] = 20(0.9048)^n\,\mathrm{u}[n]$

$\mathrm{H}(z) = 20\frac{z}{z-0.9048}$ $\qquad$ $\mathrm{H}_{-1}(s) = \frac{20}{s(s+20)}$

$\mathrm{h}_{-1}(t) = 20\left(1-e^{-20t}\right)\mathrm{u}(t)$ $\qquad$ $\mathrm{H}_{-1}(z) = 20\frac{z^2}{(z-1)(z-0.9048)}$

$$\mathrm{H}(s) = \frac{20}{s+20} \Rightarrow \mathrm{h}(t) = 20e^{-20t}\,\mathrm{u}(t) \Rightarrow \mathrm{h}[n] = 20e^{-20n/200}\,\mathrm{u}[n] = 20(0.9048)^n\,\mathrm{u}[n]$$

$$\mathrm{H}_{-1}(s) = \frac{20}{s(s+20)} \Rightarrow \mathrm{h}_{-1}(t) = 20\left(1-e^{-20t}\right)\mathrm{u}(t)$$

$$\mathrm{H}(z) = 20\frac{z}{z-0.9048} \Rightarrow \mathrm{H}_{-1}(z) = 20\frac{z}{z-1}\frac{z}{z-0.9048} = 20\frac{z^2}{z^2-1.9048z+0.9048}$$

$$\mathrm{H}_{-1}(z) = 20\frac{z^2}{z^2-1.9048z+0.9048} = 20z \times \frac{z}{z^2-1.9048z+0.9048}$$

$$\mathrm{H}_{-1}(z) = \frac{20z^2}{(z-1)(z-0.9048)} \Rightarrow \lim_{n\to\infty}\mathrm{h}_{-1}[n] = \lim_{z\to1}(z-1)\frac{20z^2}{(z-1)(z-0.9048)}$$

$$= \lim_{z\to1}\frac{20z^2}{(z-0.9048)} = 210.1$$

What is the numerical final value of the digital filter to a unit sequence? <u>210.1</u>

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3. The device which accepts an impulse excitation, produces a rectangular response and is used in modeling the action of a DAC is called a zero-order hold.

4. When trying to design an approximation to an ideal differentiator ($\mathrm{y}(t)=\frac{d}{dt}(\mathrm{x}(t))$) using the step invariant technique we encounter a major problem.

$$\mathrm{Y}(s)=s\,\mathrm{X}(s)\Rightarrow \mathrm{H}(s)=s\Rightarrow \mathrm{H}_{-1}(s)=s\times 1/s=1\Rightarrow \mathrm{h}_{-1}(t)=\delta(t)$$

The next step would be to sample the CT step response. But, since the step response is a CT impulse it cannot be sampled.

5. A lowpass analog filter has unity gain at a frequency of zero and a single real pole at $s=-30$. A digital filter is designed to approximate it using the impulse-invariant technique with a sampling rate of 200 Hz. Find the following.

$\mathrm{h}(t)=30e^{-30t}\,\mathrm{u}(t)$ $\qquad$ $\mathrm{h}[n]=30(0.8607)^n\,\mathrm{u}[n]$

$\mathrm{H}(z)=30\frac{z}{z-0.8607}$ $\qquad$ $\mathrm{H}_{-1}(s)=\frac{30}{s(s+30)}$

$\mathrm{h}_{-1}(t)=30\left(1-e^{-30t}\right)\mathrm{u}(t)$ $\qquad$ $\mathrm{H}_{-1}(z)=30\frac{z^2}{(z-1)(z-0.8607)}$

$$\mathrm{H}(s)=\frac{30}{s+30}\Rightarrow \mathrm{h}(t)=30e^{-30t}\,\mathrm{u}(t)\Rightarrow \mathrm{h}[n]=30e^{-30n/200}\,\mathrm{u}[n]=30(0.8607)^n\,\mathrm{u}[n]$$

$$\mathrm{H}_{-1}(s)=\frac{30}{s(s+30)}\Rightarrow \mathrm{h}_{-1}(t)=30\left(1-e^{-30t}\right)\mathrm{u}(t)$$

$$\mathrm{H}(z)=30\frac{z}{z-0.8607}\Rightarrow \mathrm{H}_{-1}(z)=30\frac{z}{z-1}\frac{z}{z-0.8607}=30\frac{z^2}{z^2-1.8607z+0.8607}$$

$$\mathrm{H}_{-1}(z)=30\frac{z^2}{z^2-1.8607z+0.8607}=30z\times\frac{z}{z^2-1.8607z+0.8607}$$

$$\mathrm{H}_{-1}(z)=\frac{30z^2}{(z-1)(z-0.8607)}\Rightarrow \lim_{n\to\infty}\mathrm{h}_{-1}[n]=\lim_{z\to 1}(z-1)\frac{30z^2}{(z-1)(z-0.8607)}$$

$$=\lim_{z\to 1}\frac{30z^2}{(z-0.8607)}=215.36$$

What is the numerical final value of the digital filter to a unit sequence? 215.36