

Solution to ECE Test #13 S09

1. What is unique, among digital-filter design techniques we have covered, about the way the bilinear technique maps the s and z planes to each other?

The mapping is from each point in s to a unique point in z and vice-versa.

2. (a) Which digital-filter design technique, among the techniques we have covered, can transform a stable s -domain system into an unstable z -domain system?

The difference equation or finite-difference method.

(b) If an unstable filter is formed by this technique what one thing can be changed to make it stable and how should it be changed?

The sampling rate should be increased.

3. What generally happens to the poles and zeros of a z -domain system which approximates an s -domain system when the rate at which the CT signals are sampled is increased?

The poles and zeros all move closer to the $z = 1$ point.

4. Which of the digital-filter techniques we have covered maps every point in the range $0 < \omega < \infty$ in continuous-time into a corresponding unique point in the range $0 < \Omega < \pi$ in discrete time and vice-versa?

The bilinear technique.

5. A digital filter designed using the direct substitution method has zeros at $z = e^{\pm j\pi/3}$. If the same design technique is used, except that the sampling rate is decreased by a factor of 2, what are the new locations of these zeros?

The mapping of zeros from s to z is $s - z_0 \rightarrow z - e^{z_0 T_s}$ where z_0 is a zero location in the s plane. If f_s is decreased by a factor of 2 that means T_s is increased by a factor of 2 and putting the new zero locations at $e^{2z_0 T_s} = e^{\pm j2\pi/3}$.

6. A digital filter designed using the direct substitution method has a pole at $z = 0.5$. If the same design technique is used, except that the sampling rate is increased by a factor of 2, what is the new location of this pole?

The mapping of poles from s to z is $s - p_0 \rightarrow z - e^{p_0 T_s}$ where p_0 is a pole location in the s plane. If f_s is increased by a factor of 2 that means T_s is decreased by a factor of 2 putting the new z -plane pole at $e^{p_0 (T_s/2)} = \sqrt{e^{p_0 T_s}} = \sqrt{0.5} = 0.707$.

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The mapping of zeros from s to z is $s - z_0 \rightarrow z - e^{z_0 T_s}$ where z_0 is a zero location in the s plane. If f_s is increased by a factor of 2 that means T_s is decreased by a factor of 2 and putting the new zero locations at $e^{z_0 T_s / 2} = e^{\pm j\pi/6}$.

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