Solution to ECE Test #8 S07 #1

1. Find the numerical locations of all the finite poles and zeros of a first-order ($n = 1$) bandpass Butterworth filter with cutoff frequencies of 20 Hz and 30 Hz.

$$H_{LP}(s) = \frac{1}{s+1}$$

$$H_{BP}(s) = H_{LP}(s)\Big|_{s \to \frac{s^2+\omega_L\omega_H}{s(\omega_H-\omega_L)}} = \frac{1}{\frac{s^2+\omega_L\omega_H}{s(\omega_H-\omega_L)}+1}$$

$$H_{BP}(s) = \frac{s(\omega_H-\omega_L)}{s^2+\omega_L\omega_H+s(\omega_H-\omega_L)}$$

$$\omega_L = 20\times 2\pi = 40\pi = 125.66 \ , \ \omega_H = 30\times 2\pi = 60\pi = 188.5$$

$$H_{BP}(s) = \frac{20\pi s}{s^2+20\pi s+2400\pi^2} = 20\pi\frac{s}{(s+10\pi-j47.96\pi)(s+10\pi+j47.96\pi)}$$

Two finite poles at $s = -10\pi \pm j47.96\pi$ or $s = -31.42 \pm 150.67$ or $153.91e^{\pm j1.776}$
One finite zero at $s = 0$

2. How many <u>finite</u> poles and zeros do the following Butterworth filters have?

Second-order ($n = 2$) bandstop # Poles <u>4</u> # Zeros <u>4</u>

Third-order ($n = 3$) highpass # Poles <u>3</u> # Zeros <u>3</u>

Solution to ECE Test #8 S07 #2

1. Find the numerical locations of all the finite poles and zeros of a first-order ($n = 1$) bandstop Butterworth filter with cutoff frequencies of 20 Hz and 30 Hz.

$$H_{LP}(s) = \frac{1}{s+1}$$

$$H_{BS}(s) = H_{LP}(s)\Big|_{s \to \frac{s(\omega_H - \omega_L)}{s^2 + \omega_L \omega_H}} = \frac{1}{\frac{s(\omega_H - \omega_L)}{s^2 + \omega_L \omega_H} + 1}$$

$$H_{BS}(s) = \frac{s^2 + \omega_L \omega_H}{s(\omega_H - \omega_L) + s^2 + \omega_L \omega_H}$$

$$\omega_L = 20 \times 2\pi = 40\pi = 125.66 \ , \ \omega_H = 30 \times 2\pi = 60\pi = 188.5$$

$$H_{BS}(s) = \frac{s^2 + 2400\pi^2}{s^2 + 20\pi s + 2400\pi^2} = \frac{s^2 + 2400\pi^2}{(s + 10\pi - j47.96\pi)(s + 10\pi + j47.96\pi)}$$

Two finite poles at $s = -10\pi \pm j47.96\pi$ or $s = -31.42 \pm 150.67$ or $153.91 e^{\pm j1.776}$
Two finite zeros at $s = \pm j153.91$

2. How many <u>finite</u> poles and zeros do the following Butterworth filters have?

Second-order ($n = 2$) bandpass # Poles <u>4</u> # Zeros <u>2</u>

Third-order ($n = 3$) highpass # Poles <u>3</u> # Zeros <u>3</u>

Solution to ECE Test #8 S07 #3

1. Find the numerical locations of all the finite poles and zeros of a first-order ($n = 1$) bandstop Butterworth filter with cutoff frequencies of 30 Hz and 50 Hz.

$$H_{LP}(s) = \frac{1}{s+1}$$

$$H_{BS}(s) = H_{LP}(s)\Big|_{s \to \frac{s(\omega_H - \omega_L)}{s^2 + \omega_L \omega_H}} = \frac{1}{\frac{s(\omega_H - \omega_L)}{s^2 + \omega_L \omega_H} + 1}$$

$$H_{BS}(s) = \frac{s^2 + \omega_L \omega_H}{s(\omega_H - \omega_L) + s^2 + \omega_L \omega_H}$$

$$\omega_L = 30 \times 2\pi = 60\pi = 188.5 \ , \ \omega_H = 50 \times 2\pi = 100\pi = 314.17$$

$$H_{BS}(s) = \frac{s^2 + 6000\pi^2}{s^2 + 40\pi s + 6000\pi^2} = \frac{s^2 + 6000\pi^2}{(s + 20\pi - j74.83\pi)(s + 20\pi + j74.83\pi)}$$

Two finite poles at $s = -20\pi \pm j74.83\pi$ or $s = -62.83 \pm 235.09$ or $243.35e^{\pm j1.832}$
Two finite zeros at $s = \pm j243.35$

2. How many <u>finite</u> poles and zeros do the following Butterworth filters have?

Second-order ($n = 2$) bandpass # Poles <u>4</u> # Zeros <u>2</u>

Third-order ($n = 3$) highpass # Poles <u>3</u> # Zeros <u>3</u>