Solution of ECE 316 Final Examination Su03 #1

1. (1 pt) Which of the digital filter design techniques we studied can create from a stable CT system an unstable DT system? Finite difference Technique

2. (1 pt) Which of the digital filter design techniques we studied maps the s and z planes on a one-to-one basis in both directions? That is, for every s there is a unique z and for every z there is a unique s. Bilinear z Transform

3. (8 pts) Find the inverse z transform of $\mathrm{H}(z) = \dfrac{z^3}{(z-1)^2\left(z-\frac{1}{2}\right)}$.

$$\frac{\mathrm{H}(z)}{z} = \frac{z^2}{(z-1)^2\left(z-\frac{1}{2}\right)} = \frac{2}{(z-1)^2} + \frac{0}{z-1} + \frac{1}{z-\frac{1}{2}}$$

$$\mathrm{H}(z) = \frac{2z}{(z-1)^2} + \frac{z}{z-\frac{1}{2}}$$

$$\mathrm{h}[n] = \left[2n + \left(\frac{1}{2}\right)^n\right]\mathrm{u}[n]$$

Alternate Solution:

$$\mathrm{H}(z) = \frac{z^3}{(z-1)^2\left(z-\frac{1}{2}\right)} = \frac{z^3}{z^3 - \frac{5}{2}z^2 + 2z - \frac{1}{2}} = 1 + \frac{\frac{5}{2}z^2 - 2z + \frac{1}{2}}{(z-1)^2\left(z-\frac{1}{2}\right)}$$

$$\mathrm{H}(z) = 1 + \frac{2}{(z-1)^2} + \frac{2}{z-1} + \frac{\frac{1}{2}}{z-\frac{1}{2}}$$

$$\mathrm{h}[n] = \delta[n] + 2\,\mathrm{ramp}[n-1] + 2\,\mathrm{u}[n-1] + \frac{1}{2}\left(\frac{1}{2}\right)^{n-1}\mathrm{u}[n-1]$$

$$\mathrm{h}[n] = \delta[n] + \left[2(n-1) + 2 + \left(\frac{1}{2}\right)^n\right]\mathrm{u}[n-1]$$

$$\mathrm{h}[n] = \delta[n] + \left[2n + \left(\frac{1}{2}\right)^n\right]\mathrm{u}[n-1]$$

and this solution is equivalent to the previous one.

4. The s-domain transfer function, $\mathrm{H}(s) = \frac{10}{s+1}$ is approximated by digital filters using the impulse invariant, step invariant, finite difference (using backward differences), direct substitution, matched z transform and bilinear z transform techniques yielding the following 6 z-domain transfer functions. In each case the sampling rate used was 2 Hz. Identify which method was used for each one. (There are only five answers because two methods yielded identical results. Therefore one of the choices has two answers.) (4 pts each)

(a) $\mathrm{H}(z) = 2\frac{z+1}{z-0.6}$ Bilinear

$$\mathrm{H}(z) = \frac{10}{4\frac{z-1}{z+1}+1} = \frac{10(z+1)}{4(z-1)+z+1} = \frac{10(z+1)}{5z-3} = 2\frac{z+1}{z-0.6}$$

(b) $\mathrm{H}(z) = \frac{10z}{z-0.6065}$ Impulse Invariant and Matched z Transform

Impulse Invariant:

$$\mathrm{h}(t) = 10e^{-t}\,\mathrm{u}(t) \Rightarrow \mathrm{h}[n] = 10e^{-\frac{n}{2}}\,\mathrm{u}[n] = 10(0.6065)\,\mathrm{u}[n] \Rightarrow \mathrm{H}(z) = \frac{10z}{z-0,6065}$$

Matched z Transform:

$$\mathrm{H}(z) = \frac{10}{1-e^{-\frac{1}{2}}z^{-1}} = \frac{10}{1-0.6065z^{-1}} = \frac{10z}{z-0.6065}$$

(c) $\mathrm{H}(z) = \frac{3.333z}{z-0.667}$ Finite Difference

$$\mathrm{H}(z) = \frac{10}{\frac{1-z^{-1}}{0.5}+1} = \frac{5}{1-z^{-1}+0.5} = \frac{5}{1.5-z^{-1}} = \frac{5z}{1.5z-1} = \frac{3.333z}{z-0.667}$$

(d) $\mathrm{H}(z) = \frac{3.935}{z-0.6065}$ Step Invariant

$$\mathrm{h}_{-1}(t) = 10\left(1-e^{-t}\right)\mathrm{u}(t) \Rightarrow \mathrm{h}_{-1}[n] = 10\left(1-e^{-\frac{n}{2}}\right)\mathrm{u}[n]$$

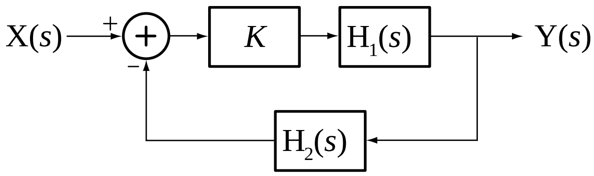
$$\mathrm{H}_{-1}(z) = 10\left(\frac{z}{z-1} - \frac{z}{z-0.6065}\right) \Rightarrow \mathrm{H}(z) = 10\left(1 - \frac{z-1}{z-0.6065}\right) = \frac{3.935}{z-0.6065}$$

(e) $\mathrm{H}(z) = \frac{10}{z-0.6065}$ Direct Substitution

$$H(z) = \frac{10}{z - e^{-\frac{1}{2}}} = \frac{10}{z - 0.6065}$$

5. (3 pts) A root locus begins on the poles of the loop transfer function and ends on the zeros of that same function. (Loop transfer function, not just transfer function.)

6. (5 pts each) For each combination of $H_1(s)$ and $H_2(s)$ sketch a root locus and determine whether this CT feedback system can be unstable for a finite positive value of K.



(a) $H_1(s) = \frac{s}{s + \frac{1}{2}}$, $H_2(s) = 1$ Cannot be unstable

Root locus is a straight line from $-\frac{1}{2}$ to 0 in the s plane.

(b) $H_1(s) = \frac{s - \frac{1}{2}}{s + \frac{1}{2}}$, $H_2(s) = 1$ Can be unstable

Root locus is a straight line from $-\frac{1}{2}$ to $\frac{1}{2}$ in the s plane.

(c) $H_1(s) = \frac{s - \frac{1}{4}}{s^2 + \frac{1}{2}s + \frac{5}{16}}$, $H_2(s) = 1$ Can be unstable

Root locus begins on two complex conjugate poles in the left half of the s plane, both locii go to the real axis and then one goes to the zero at $\frac{1}{2}$ on the real axis (in the right half-plane) and the other goes to negative infinity on the real axis.

7. (5 pts each) In each part of the previous problem, replace s by z in the transfer functions and in the system diagram and draw a root locus and answer the question of stability for a finite positive value of K again for those DT feedback systems.

(a) $H_1(z) = \frac{z}{z + \frac{1}{2}}$, $H_2(z) = 1$ Cannot be unstable

Root locus is a straight line from $-\frac{1}{2}$ to 0 in the z plane.

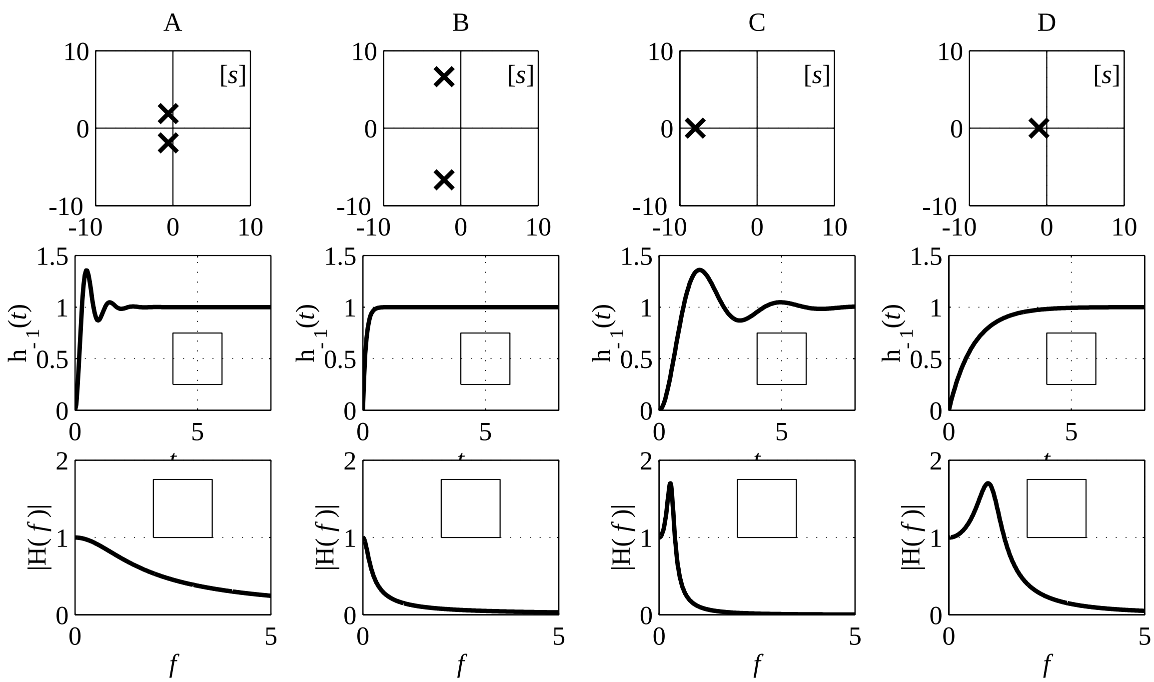
(b) $\quad \mathrm{H}_1(z) = \dfrac{z - \frac{1}{2}}{z + \frac{1}{2}}, \ \mathrm{H}_2(z) = 1 \qquad$ Cannot be unstable

Root locus is a straight line from $-\frac{1}{2}$ to $\frac{1}{2}$ in the z plane.

(c) $\quad \mathrm{H}_1(z) = \dfrac{z - \frac{1}{4}}{z^2 + \frac{1}{2}z + \frac{5}{16}}, \ \mathrm{H}_2(z) = 1 \qquad$ Can be unstable

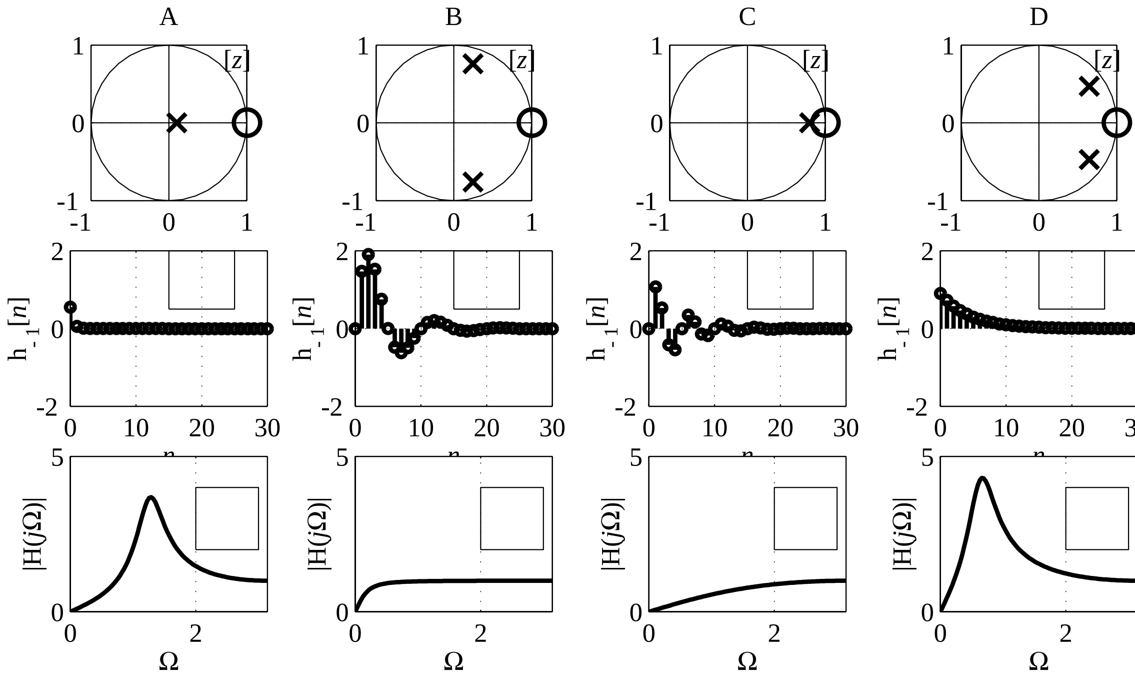
Root locus begins on two complex conjugate poles inside the unit circle of the z plane, both locii go to the real axis and then one goes to the zero at $\frac{1}{2}$ on the real axis (in inside the unit circle) and the other goes to negative infinity on the real axis (outside the unit circle).

8. (3 pts each) Match the system pole-zero diagrams to the step responses and frequency responses by writing the appropriate letter in the boxes provided.



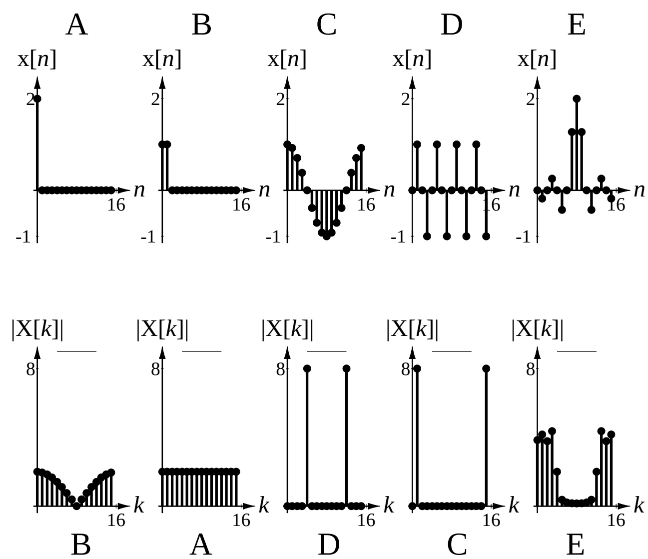
Step Responses: BCAD
Frequency Responses: CDAB

9. (3 pts each) Match the system pole-zero diagrams to the step responses and frequency responses by writing the appropriate letter in the boxes provided.



Unit-Sequence Responses: ADBC
Frequency Responses: BCAD

10. (3 pts each) Match DT functions to their DFT magnitudes by writing the appropriate identifying letter in the blank spaces provided.



11. (1 pt) A signal can be simultaneously unlimited in time and unlimited in frequency.

<u>True</u>

12. (2 pts) A cosine, $\mathrm{x}(t)$, and a sine signal, $\mathrm{y}(t)$, of the same frequency are added to form a composite signal, $\mathrm{z}(t)$. The signal, $\mathrm{z}(t)$, is then sampled at exactly its Nyquist rate with the usual assumption that a sample occurs at time, $t = 0$. Which of the two signals, $\mathrm{x}(t)$ or $\mathrm{y}(t)$, would, if sampled by itself, produce exactly the same set of samples?

$\mathrm{x}(t)$

13. (2 pts) Only a bandlimited, periodic signal can be completely described by a finite set of samples taken from it.

14. (1 pt) The fast Fourier transform is an efficient algorithm for computing the discrete Fourier transform (DFT).

15. (1 pt) The fast Fourier transform only achieves its maximum efficiency if the number of samples is an integer power of 2.