Solution of ECE 316 Final Examination S11

1. Find the numerical values of the constants.

(a) $4e^{-at}\,\mathrm{u}(t) * Ae^{-t/2}\,\mathrm{u}(t) \xleftrightarrow{\mathcal{L}} \dfrac{36}{s^2+bs+3}$, $\sigma > -1/2 \cap \sigma > -a$

$$4e^{-at}\,\mathrm{u}(t) * Ae^{-t/2}\,\mathrm{u}(t) \xleftrightarrow{\mathcal{L}} \frac{4}{s+a}\frac{A}{s+1/2} = \frac{4A}{s^2+(a+1/2)s+a/2} = \frac{36}{s^2+bs+3} \quad , \ \sigma > 0$$

$4A = 36 \Rightarrow A = 9$, $a/2 = 3 \Rightarrow a = 6$, $a + 1/2 = b \Rightarrow b = 6.5$

(b) $12\alpha^{n-1}\,\mathrm{u}[n] \xleftrightarrow{\mathcal{Z}} \dfrac{36z}{z-\alpha}$, $|z| > |\alpha|$

$$12\alpha^{n-1}\,\mathrm{u}[n] = 12\alpha^{-1}\alpha^{n}\,\mathrm{u}[n] \xleftrightarrow{\mathcal{Z}} 12\alpha^{-1}\frac{z}{z-\alpha} = \frac{36z}{z-\alpha} \quad , \ |z| > 1$$

$$12\alpha^{-1} = 36 \Rightarrow \alpha = \frac{12}{36} \Rightarrow \alpha = 0.3333$$

2. If $\mathrm{H}(s) = \dfrac{A}{s(s+a)(s+b)}$ is the transfer function of a physically-realizable system and $a = 4e^{-j\pi/3}$ what is the numerical value of b?

Since complex poles must occur in complex-conjugate pairs for a physically-realizable system, $b = 4e^{j\pi/3}$ or $2 + j3.4641$.

3. Circle the signals that are bandlimited.

$\boxed{\mathrm{tri}(t)*\mathrm{sinc}(t)}$	$\mathrm{tri}(t)\mathrm{sinc}(t)$	$\mathrm{rect}(t)*\delta_2(t)$	$\boxed{\mathrm{sinc}(t)*\delta_2(t)}$
$\mathrm{sinc}(t)\delta_2(t)$	e^{-t^2}	$\boxed{\sin(t)+\cos(t)}$	$\mathrm{rect}(t)+\mathrm{sinc}(t)$
$\mathrm{tri}(t)+\cos(t)$	$\boxed{\mathrm{sinc}(t)+\cos(t)}$	$\boxed{\mathrm{sinc}(t)*\cos(t)}$	$\boxed{\mathrm{sinc}(t)\cos(t)}$

4. Let $\{\mathrm{x}[0],\mathrm{x}[1],\mathrm{x}[2],\mathrm{x}[3]\}\xleftrightarrow[4]{\mathcal{DFT}}\{\mathrm{X}[0],\mathrm{X}[1],\mathrm{X}[2],\mathrm{X}[3]\}$. If the x values represent one period of a periodic signal, the average value of the signal is $\frac{\mathrm{x}[0]+\mathrm{x}[1]+\mathrm{x}[2]+\mathrm{x}[3]}{4}$. What is the average value of the signal expressed solely in terms of its DFT $\mathrm{X}[k]=\{\mathrm{X}[0],\mathrm{X}[1],\mathrm{X}[2],\mathrm{X}[3]\}$?

Average value is $\mathrm{X}[0]/4$

5. Circle the descriptions that apply to systems that are stable.

$\mathrm{H}(s)=\frac{13}{s(s+4)}$

$\boxed{\text{Unity-gain feedback system with } \mathrm{H}_1(s)=\frac{5}{s^2+3}}$.

System with impulse response $\mathrm{h}(t)=7\cos(22\pi t)\mathrm{u}(t)$.

$\boxed{\text{System with zeros at } s=2 \text{ and } s=-0.5 \text{ and poles at } s=-0.8 \text{ and } s=-1.8.}$

System with zeros at $z=2$ and $z=-0.5$ and poles at $z=-0.8$ and $z=-1.8$.

System with impulse response $\mathrm{h}[n]=-8(-1.2)^n\cos(13n/25)\mathrm{u}[n]$.

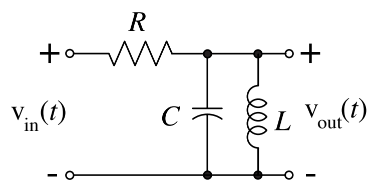
6. Of the systems with these two transfer functions, circle the one whose step response approaches its final value faster.

(a) $H_1(s) = \frac{1}{s+4}$ $\boxed{H_2(s) = \frac{1}{s+10}}$

(b) $\boxed{H_1(z) = \frac{z}{z+0.2}}$ $H_2(z) = \frac{z}{z+0.5}$

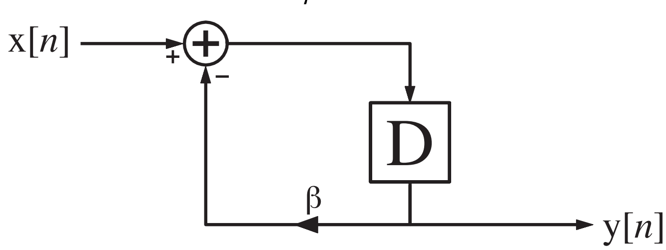
7. For the practical passive filter below with transfer function $H(s) = V_{out}(s)/V_{in}(s)$ what is the numerical slope, in dB per decade, of a magnitude Bode diagram of its frequency response at frequencies approaching zero and at frequencies approaching infinity?

For $f \to 0$, slope is 20 dB/decade
For $f \to \infty$, slope is -20 dB/decade



8. For the practical discrete-time filter below with transfer function $H(z) = Y(z)/X(z)$ what are the numerical values of its frequency response at $\Omega = 0$ and at $\Omega = \pi$?

$\beta = 0.7$



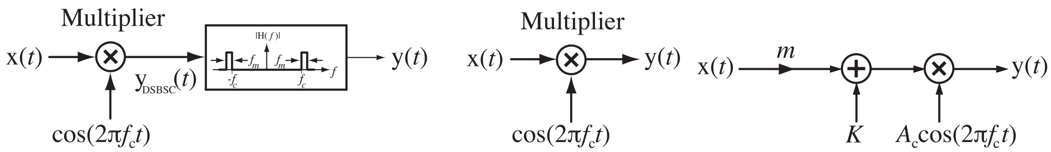
$$H(z) = \frac{1}{z+0.7} \Rightarrow H(e^{j\Omega}) = \frac{1}{e^{j\Omega}+0.7}$$

$$H(e^{j0}) = \frac{1}{1+0.7} = 0.5882$$

$$H(e^{j\pi}) = \frac{1}{-1+0.7} = -3.333$$

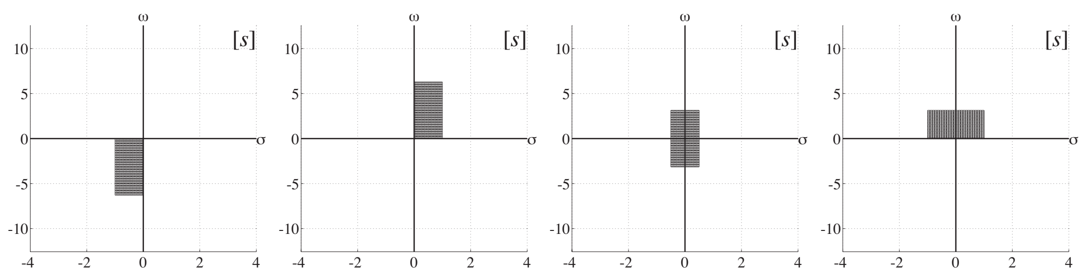
9. The three modulators below are the three amplitude modulation systems we have studied. Label them (in the blank spaces provided) as DSBSC, DSBTC or SSBSC.

SSBSC DSBSC DSBTC

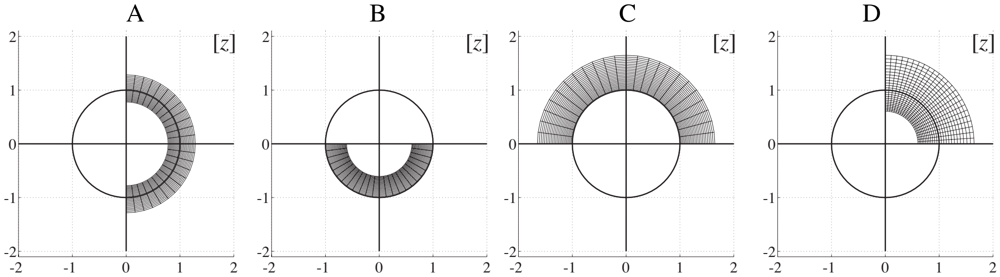


10. Match the s-plane regions to the z-plane regions by entering in the blank space provided above each s-plane region the letter designation of the corresponding z-plane region. (The mapping is done using $z = e^{sT_s}$ with $T_s = 0.5$).

B C A D



A B C D



11. In a discrete-time feedback system with loop transfer function $\mathrm{T}(z)$ and an adjustable gain factor K, what relationship between the number of zeros and the number of poles of $\mathrm{T}(z)$ guarantees that the system will become unstable at a finite positive value of K?

    If the number of poles exceeds the number of zeros, the root locus must leave the unit circle to approach a zero at infinity and, when that happens, the system becomes unstable.

12. Which digital filter design method can produce a filter with a linear phase shift as a function of frequency for any arbitrary filter order?

    FIR (Bessel filters are optimized for phase linearity but their phases are still not perfectly linear.)

13. Is an envelope detector an asynchronous or synchronous demodulation technique?

    $\boxed{\text{Asynchronous}}$ Synchronous

14. What is the main reason that transmitted-carrier modulation is used in commercial radio instead of suppressed-carrier modulation?

    DSBTC modulation can be demodulated by a simple, cheap envelope detector instead of a more complicated and expensive synchronous detector.

Solution of ECE 316 Final Examination S11

1. Find the numerical values of the constants.

(a) $4e^{-at}\,\mathrm{u}(t) * Ae^{-t/3}\,\mathrm{u}(t) \xleftrightarrow{\mathcal{L}} \dfrac{40}{s^2+bs+3}$, $\sigma > -1/3 \cap \sigma > -a$

$$4e^{-at}\,\mathrm{u}(t) * Ae^{-t/3}\,\mathrm{u}(t) \xleftrightarrow{\mathcal{L}} \frac{4}{s+a}\frac{A}{s+1/3} = \frac{4A}{s^2+(a+1/3)s+a/3} = \frac{40}{s^2+bs+3} \quad , \ \sigma > 0$$

$4A = 40 \Rightarrow A = 10$, $a/3 = 3 \Rightarrow a = 9$, $a + 1/3 = b \Rightarrow b = 9.3333$

(b) $12\alpha^{n-1}\,\mathrm{u}[n] \xleftrightarrow{\mathcal{Z}} \dfrac{45z}{z-\alpha}$, $|z| > |\alpha|$

$$12\alpha^{n-1}\,\mathrm{u}[n] = 12\alpha^{-1}\alpha^{n}\,\mathrm{u}[n] \xleftrightarrow{\mathcal{Z}} 12\alpha^{-1}\frac{z}{z-\alpha} = \frac{45z}{z-\alpha} \quad , \ |z| > 1$$

$$12\alpha^{-1} = 45 \Rightarrow \alpha = \frac{12}{45} \Rightarrow \alpha = 0.2667$$

2. If $\mathrm{H}(s) = \dfrac{A}{s(s+a)(s+b)}$ is the transfer function of a physically-realizable system and $a = 7e^{-j\pi/4}$ what is the numerical value of b?

Since complex poles must occur in complex-conjugate pairs for a physically-realizable system, $b = 7e^{j\pi/4}$ or $4.9497 + j4.9497$.

3. Circle the signals that are bandlimited.

$\mathrm{rect}(t)+\mathrm{sinc}(t)$	$\mathrm{tri}(t)\mathrm{sinc}(t)$	$\boxed{\mathrm{sinc}(t)*\delta_2(t)}$	$\boxed{\sin(t)+\cos(t)}$
$\boxed{\mathrm{sinc}(t)\cos(t)}$	$\boxed{\mathrm{sinc}(t)*\cos(t)}$	$\boxed{\mathrm{tri}(t)*\mathrm{sinc}(t)}$	$\boxed{\mathrm{sinc}(t)+\cos(t)}$
$\mathrm{tri}(t)+\cos(t)$	$\mathrm{sinc}(t)\delta_2(t)$	e^{-t^2}	$\mathrm{rect}(t)*\delta_2(t)$

4. Let $\{x[0],x[1],x[2],x[3]\} \xleftrightarrow[4]{\mathcal{DFT}} \{X[0],X[1],X[2],X[3]\}$. If the x values represent one period of a periodic signal, the average value of the signal is $\dfrac{x[0]+x[1]+x[2]+x[3]}{4}$. What is the average value of the signal expressed solely in terms of its DFT $X[k]=\{X[0],X[1],X[2],X[3]\}$?

Average value is $X[0]/4$

5. Circle the descriptions that apply to systems that are stable.

$\boxed{H(s)=\dfrac{13}{(s+3)(s+4)}}$

Unity-gain feedback system with $H_1(s)=\dfrac{5}{s^2(s+3)}$.

System with impulse response $h[n]=(-1)^n u[n]$.

System with zeros at $s=2$ and $s=-0.5$ and poles at $s=-0.8$ and $s=0.8$.

$\boxed{\text{System with zeros at } z=2 \text{ and } z=-0.5 \text{ and poles at } z=-0.8 \text{ and } z=0.8.}$

System with impulse response $h(t)=13e^{1.2t}\cos(13\pi t)u(t)$.

6. Of the systems with these two transfer functions, circle the one whose step response approaches its final value faster.

(a) $\boxed{H_1(z) = \frac{z}{z+0.2}}$ $\quad H_2(z) = \frac{z}{z+0.5}$

(b) $H_1(s) = \frac{1}{s+4}$ $\quad \boxed{H_2(s) = \frac{1}{s+10}}$

7. For the practical passive filter below with transfer function $H(s) = V_{out}(s)/V_{in}(s)$ what is the numerical slope, in dB per decade, of a magnitude Bode diagram of its frequency response at frequencies approaching zero and at frequencies approaching infinity?

For $f \to 0$, slope is 20 dB/decade
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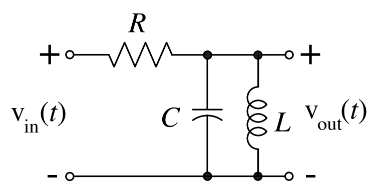
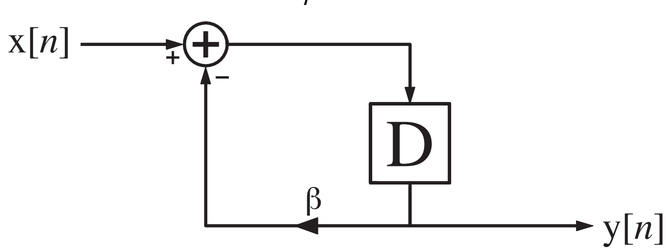


8. For the practical discrete-time filter below with transfer function $H(z) = Y(z)/X(z)$ what are the numerical values of its frequency response at $\Omega = 0$ and at $\Omega = \pi$?

$\beta = 0.5$



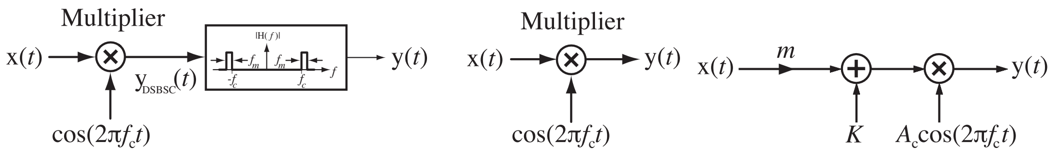
$$H(z) = \frac{1}{z+0.5} \Rightarrow H(e^{j\Omega}) = \frac{1}{e^{j\Omega}+0.5}$$

$$H(e^{j0}) = \frac{1}{1+0.5} = 0.6667$$

$$H(e^{j\pi}) = \frac{1}{-1+0.5} = -2$$

9. The three modulators below are the three amplitude modulation systems we have studied. Label them (in the blank spaces provided) as DSBSC, DSBTC or SSBSC.

SSBSC DSBSC DSBTC



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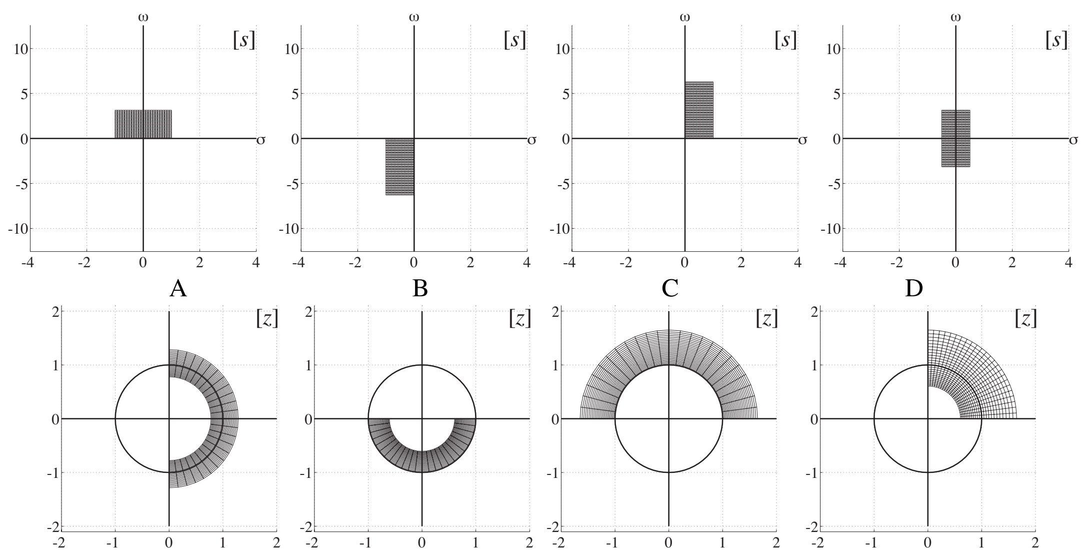
D B C A



11. In a discrete-time feedback system with loop transfer function $T(z)$ and an adjustable gain factor K, what relationship between the number of zeros and the number of poles of $T(z)$ guarantees that the system will become unstable at a finite positive value of K?

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$$4e^{-at}\,\mathrm{u}(t) * Ae^{-t/4}\,\mathrm{u}(t) \xleftrightarrow{\mathcal{L}} \frac{4}{s+a}\frac{A}{s+1/4} = \frac{4A}{s^2+(a+1/4)s+a/4} = \frac{20}{s^2+bs+3} \quad , \ \sigma > 0$$

$4A = 20 \Rightarrow A = 5$, $a/4 = 3 \Rightarrow a = 12$, $a + 1/4 = b \Rightarrow b = 12.25$

(b) $12\alpha^{n-1}\,\mathrm{u}[n] \xleftrightarrow{\mathcal{Z}} \dfrac{16z}{z-\alpha}$, $|z| > |\alpha|$

$$12\alpha^{n-1}\,\mathrm{u}[n] = 12\alpha^{-1}\alpha^{n}\,\mathrm{u}[n] \xleftrightarrow{\mathcal{Z}} 12\alpha^{-1}\frac{z}{z-\alpha} = \frac{16z}{z-\alpha} \quad , \ |z| > 1$$

$$12\alpha^{-1} = 16 \Rightarrow \alpha = \frac{12}{16} \Rightarrow \alpha = 0.75$$

2. If $\mathrm{H}(s) = \dfrac{A}{s(s+a)(s+b)}$ is the transfer function of a physically-realizable system and $a = 9e^{-j\pi/2}$ what is the numerical value of b?

Since complex poles must occur in complex-conjugate pairs for a physically-realizable system, $b = 9e^{j\pi/2}$ or $j9$.

3. Circle the signals that are bandlimited.

$$e^{-t^2} \qquad \boxed{\mathrm{sinc}(t)\cos(t)} \qquad \mathrm{tri}(t)+\cos(t) \qquad \boxed{\mathrm{sinc}(t)*\cos(t)}$$

$$\boxed{\sin(t)+\cos(t)} \qquad \boxed{\mathrm{tri}(t)*\mathrm{sinc}(t)} \qquad \boxed{\mathrm{sinc}(t)+\cos(t)} \qquad \mathrm{rect}(t)+\mathrm{sinc}(t)$$

$$\boxed{\mathrm{sinc}(t)*\delta_2(t)} \qquad \mathrm{sinc}(t)\delta_2(t) \qquad \mathrm{rect}(t)*\delta_2(t) \qquad \mathrm{tri}(t)\mathrm{sinc}(t)$$

4. Let $\{\mathrm{x}[0],\mathrm{x}[1],\mathrm{x}[2],\mathrm{x}[3]\} \xleftrightarrow[4]{\mathcal{DFT}} \{\mathrm{X}[0],\mathrm{X}[1],\mathrm{X}[2],\mathrm{X}[3]\}$. If the x values represent one period of a periodic signal, the average value of the signal is $\dfrac{\mathrm{x}[0]+\mathrm{x}[1]+\mathrm{x}[2]+\mathrm{x}[3]}{4}$. What is the average value of the signal expressed solely in terms of its DFT $\mathrm{X}[k]=\{\mathrm{X}[0],\mathrm{X}[1],\mathrm{X}[2],\mathrm{X}[3]\}$?

Average value is $\mathrm{X}[0]/4$

5. Circle the descriptions that apply to systems that are stable.

$\boxed{\text{System with impulse response } \mathrm{h}[n]=-8(-0.8)^n\cos(13n/25)\mathrm{u}[n]}$.

$\mathrm{H}(s)=\dfrac{13}{s(s+4)}$

$\boxed{\text{Unity-gain feedback system with } \mathrm{H}_1(s)=\dfrac{5}{s^2+3}}$.

System with impulse response $\mathrm{h}(t)=7\cos(22\pi t)\mathrm{u}(t)$.

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System with zeros at $z=2$ and $z=-0.5$ and poles at $z=-0.8$ and $z=-1.8$.

6. Of the systems with these two transfer functions, circle the one whose step response approaches its final value faster.

(a) $\mathrm{H}_1(s) = \dfrac{1}{s+4}$ $\qquad \boxed{\mathrm{H}_2(s) = \dfrac{1}{s+10}}$

(b) $\boxed{\mathrm{H}_1(z) = \dfrac{z}{z+0.2}}$ $\qquad \mathrm{H}_2(z) = \dfrac{z}{z+0.5}$

7. For the practical passive filter below with transfer function $\mathrm{H}(s) = \mathrm{V}_{out}(s)/\mathrm{V}_{in}(s)$ what is the numerical slope, in dB per decade, of a magnitude Bode diagram of its frequency response at frequencies approaching zero and at frequencies approaching infinity?

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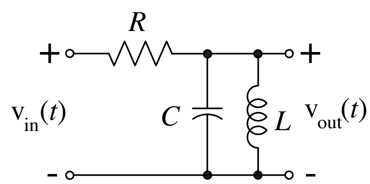
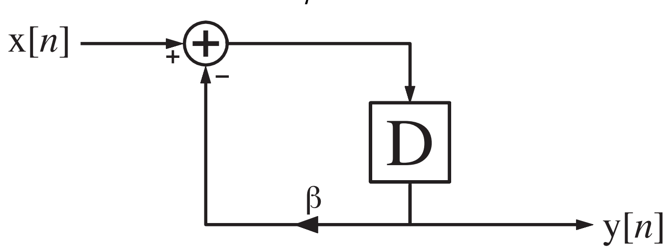


8. For the practical discrete-time filter below with transfer function $\mathrm{H}(z) = \mathrm{Y}(z)/\mathrm{X}(z)$ what are the numerical values of its frequency response at $\Omega = 0$ and at $\Omega = \pi$?

$\beta = 0.3$



$$\mathrm{H}(z) = \frac{1}{z+0.3} \Rightarrow \mathrm{H}\left(e^{j\Omega}\right) = \frac{1}{e^{j\Omega}+0.3}$$
$$\mathrm{H}\left(e^{j0}\right) = \frac{1}{1+0.3} = 0.7692$$
$$\mathrm{H}\left(e^{j\pi}\right) = \frac{1}{-1+0.3} = -1.4286$$

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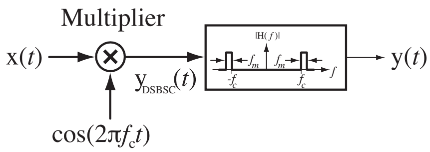
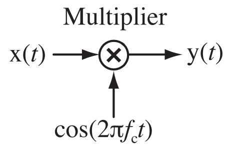
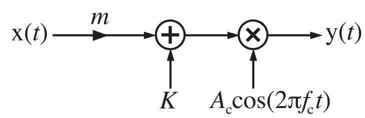
SSBSC DSBSC DSBTC







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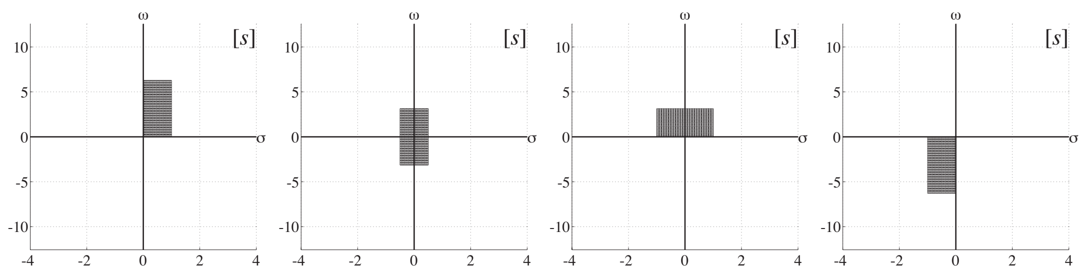
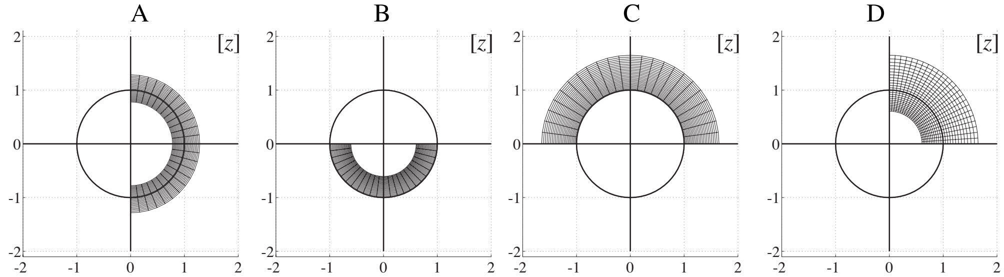
C A D B



A B C D



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