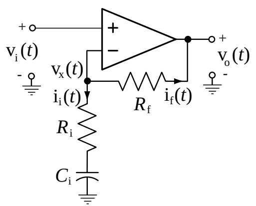
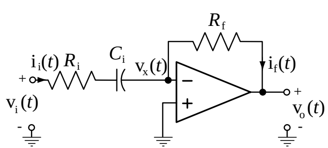
Solution to ECE 315 Final Examination Su05

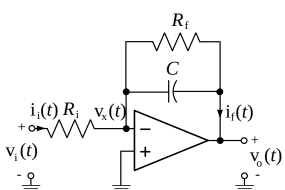
1. For each active filter below find the numerical value of the magnitude of the transfer function $\mathrm{H}(f) = V_o(f)/V_i(f)$ at the two extremes, $f = 0$ and $f \to +\infty$. (All resistors are 1 ohm and all capacitors are 1 F.)

(a) 

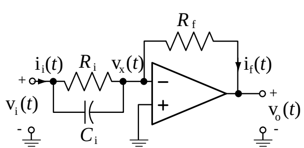
$|\mathrm{H}(0)| = 1$ or 0 dB $\qquad |\mathrm{H}(+\infty)| = 2$

(b) 

$|\mathrm{H}(0)| = 0$ or $-\infty$ dB $\qquad |\mathrm{H}(+\infty)| = |-1| = 1$ or 0 dB

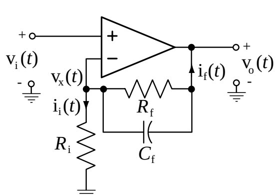
(c) 

$|\mathrm{H}(0)| = |-1| = 1$ or 0 dB $\qquad |\mathrm{H}(+\infty)| = 0$ or $-\infty$ dB

(d) 

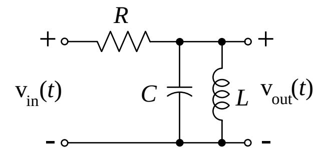
$|\mathrm{H}(0)| = |-1| = 1$ or 0 dB

$|\mathrm{H}(+\infty)| = |-\infty| = \infty$ or $+\infty$ dB

(e) 

$|\mathrm{H}(0)| = 2$ or 6 dB $\qquad |\mathrm{H}(+\infty)| = 1$ or 0 dB

2. In the circuit below $R = 100\ \Omega$, $C = 5\ \mu\mathrm{F}$, $L = 10\ \mathrm{mH}$.



(a) Find the frequency at which the magnitude of the transfer function is a maximum.

$$\mathrm{H}(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)} = \frac{(j\omega L / j\omega C)/(j\omega L + 1/j\omega C)}{(j\omega L / j\omega C)/(j\omega L + 1/j\omega C) + R} = \frac{(j\omega L / j\omega C)}{(j\omega L / j\omega C) + R(j\omega L + 1/j\omega C)}$$

$$\mathrm{H}(j\omega) = \frac{j\omega L}{(j\omega)^2 RLC + j\omega L + R} = \frac{j\omega}{RC}\frac{1}{(j\omega)^2 + j\omega / RC + 1/LC}$$

Magnitude is a maximum at resonance which is at $\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{50 \times 10^{-9}}} = 4472$ or $f = 711.8$.

(b) Find the non-zero frequency at which the phase of the transfer function is zero.

Same answer, $\omega = 4472$ or $f = 711.8$.

(c) Find the phase shift of the transfer function just above and just below $f = 0$.

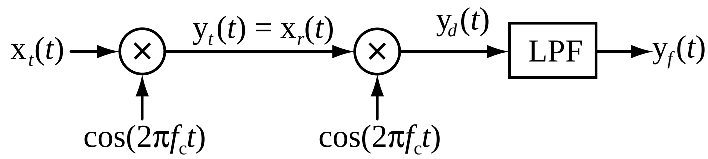
Just above $f = 0$ the phase is the phase of $\mathrm{H}(j\omega) \cong \frac{j\omega}{R} L$ which is $\pi / 2$ radians.

Just below $f = 0$ the phase is the phase of $\mathrm{H}(j\omega) \cong \frac{j\omega}{R} L$ which is $-\pi / 2$ radians.

(d) Classify this filter as a practical approximation to the ideal lowpass, highpass, bandpass or bandstop filter.

Bandpass

3. In the system below let $x_t(t) = 3\sin(1000\pi t)$, let $f_c = 5000$ and let the lowpass filter (LPF) be ideal with a transfer function magnitude of one in its passband.



(a) Using the principle that the signal power of a sum of sinusoids plus a constant is the sum of the signal power of the constant and the signal powers in the individual sinusoids <u>if their frequencies are all different</u>, find the signal power of $y_t(t)$. (The signal power of a single sinusoid is one-half of the square of its amplitude.)

$$y_t(t) = 3\sin(1000\pi t)\cos(10000\pi t)$$

$$Y_t(f) = \frac{j3}{2}\left[\delta(f+500) - \delta(f-500)\right] * \frac{1}{2}\left[\delta(f-5000) + \delta(f+5000)\right]$$

$$Y_t(f) = \frac{j3}{4}\left[\delta(f-4500) + \delta(f+5500) - \delta(f-5500) - \delta(f+4500)\right]$$

$$y_t(t) = \frac{3}{2}\left[-\sin(9000\pi t) + \sin(11000\pi t)\right]$$

$$P_y = 2 \times (3/2)^2 / 2 = \frac{9}{4} = 2.25$$

(b) Find the signal power of $y_d(t)$.

$$Y_d(f) = \frac{j3}{4}\left[\delta(f-4500) + \delta(f+5500) - \delta(f-5500) - \delta(f+4500)\right]$$
$$* \frac{1}{2}\left[\delta(f-5000) + \delta(f+5000)\right]$$

$$Y_d(f) = \frac{j3}{8}\begin{bmatrix} \delta(f-9500) + \delta(f+500) - \delta(f-10500) - \delta(f-500) \\ +\delta(f+500) + \delta(f+10500) - \delta(f-500) - \delta(f+9500) \end{bmatrix}$$

$$Y_d(f) = \frac{j3}{8}\begin{bmatrix} \delta(f-9500) - \delta(f+9500) + 2\delta(f+500) - 2\delta(f-500) \\ +\delta(f+10500) - \delta(f-10500) \end{bmatrix}$$

$$y_d(t) = \frac{3}{4}\left[-\sin(19000\pi t) + \sin(21000\pi t) + 2\sin(1000\pi t)\right]$$

$$P_y = 2\times(3/4)^2/2+(6/4)^2/2 = \frac{9}{16}+\frac{18}{16}=\frac{27}{16}=1.6875$$

(c) Find the signal power of $y_f(t)$ if the cutoff frequency of the lowpass filter is 1 kHz.

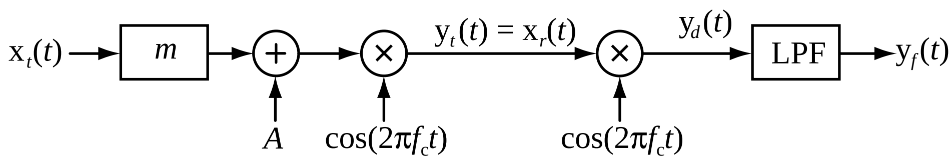
$$Y_f(f)=\frac{j3}{8}\left[2\delta(f+500)-2\delta(f-500)\right]$$

$$y_f(f)=\frac{3}{2}\sin(1000\pi t)\Rightarrow P_y=\frac{18}{16}=1.125$$

(d) Find the signal power of $y_f(t)$ if the cutoff frequency of the lowpass filter is 100 Hz.

$$P_y = 0$$

4. In the system below let $x_t(t) = 3\sin(1000\pi t)$, let $m = 1$, let $A = 3$, let $f_c = 5000$ and let the lowpass filter (LPF) be ideal with a transfer function magnitude of one in its passband.



(a) Using the principle that the signal power of a sum of sinusoids plus a constant is the sum of the signal power of the constant and the signal powers in the individual sinusoids <u>if their frequencies are all different</u>, find the signal power of $y_t(t)$. (The signal power of a single sinusoid is one-half of the square of its amplitude.)

$$y_t(t) = 3(\sin(1000\pi t) + 1)\cos(10000\pi t)$$

$$Y_t(f) = \left\{3\delta(f) + \frac{j3}{2}[\delta(f+500) - \delta(f-500)]\right\} * \frac{1}{2}[\delta(f-5000) + \delta(f+5000)]$$

$$Y_t(f) = \frac{3}{2}[\delta(f-5000) + \delta(f+5000)]$$
$$+\frac{j3}{4}[\delta(f-4500) + \delta(f+5500) - \delta(f-5500) - \delta(f+4500)]$$

$$y_t(t) = 3\cos(10000\pi t) + \frac{3}{2}[\sin(11000\pi t) - \sin(9000\pi t)]$$

$$P_y = (3^2/2) + 2\times(3/2)^2/2 = \frac{9}{2} + \frac{9}{4} = \frac{27}{4} = 6.75$$

(b) Find the signal power of $y_d(t)$.

$$Y_d(f) = \left\{\begin{array}{l}\frac{3}{2}[\delta(f-5000) + \delta(f+5000)] \\ +\frac{j3}{4}[\delta(f-4500) + \delta(f+5500) - \delta(f-5500) - \delta(f+4500)]\end{array}\right\}$$
$$* \frac{1}{2}[\delta(f-5000) + \delta(f+5000)]$$

$$Y_d(f) = \left\{ \begin{array}{l} \frac{3}{2}\left[\delta(f-5000)+\delta(f+5000)\right] * \frac{1}{2}\left[\delta(f-5000)+\delta(f+5000)\right] \\ +\frac{j3}{4}\left[\delta(f-4500)+\delta(f+5500)-\delta(f-5500)-\delta(f+4500)\right] \\ \qquad * \frac{1}{2}\left[\delta(f-5000)+\delta(f+5000)\right] \end{array} \right\}$$

$$Y_d(f) = \left\{ \begin{array}{l} \frac{3}{4}\left[\delta(f-10000)+2\delta(f)+\delta(f+10000)\right] \\ +\frac{j3}{8}\left[\begin{array}{l} \delta(f-9500)+2\delta(f+500)+\delta(f+10500) \\ -\delta(f-10500)-2\delta(f-500)-\delta(f+9500) \end{array} \right] \end{array} \right\}$$

$$y_d(t) = \frac{3}{2}\cos(20000\pi t) + \frac{3}{2} + \frac{3}{4}\left[2\sin(1000\pi t) + \sin(21000\pi t) - \sin(19000\pi t)\right]$$

$$P_y = (3/2)^2/2 + (3/2)^2 + (3/2)^2/2 + 2\times(3/4)^2/2 = \frac{9}{8} + \frac{9}{4} + \frac{9}{8} + \frac{9}{16}$$

$$P_y = \frac{18+36+18+9}{16} = \frac{81}{16} = 5.0625$$

(c) Find the signal power of $y_f(t)$ if the cutoff frequency of the lowpass filter is 1 kHz.

$$y_f(t) = \frac{3}{2} + \frac{3}{2}\sin(1000\pi t) \Rightarrow P_y = (3/2)^2 + (3/2)^2/2 = \frac{9}{4} + \frac{9}{8} = \frac{27}{8} = 3.375$$

(d) Find the signal power of $y_f(t)$ if the cutoff frequency of the lowpass filter is 100 Hz.

$$y_f(t) = \frac{3}{2} \Rightarrow P_y = (3/2)^2 = \frac{9}{4} = 2.25$$

5. What is the fundamental reason that an ideal filter cannot physically exist, even if the components in it were ideal?

Ideal filters are not causal.

6. A system is excited by a sinusoid whose signal power is 0.01 and the response is a sinusoid of the same frequency with a signal power of 4. What is the magnitude of the transfer function of the system at the frequency of the sinusoid, expressed in decibels (dB)?

Power Ratio

$$10\log_{10}\left(\frac{4}{0.01}\right) = 10\log_{10}(400) = 26 \text{ dB}$$

7. A system is excited by a sinusoid whose amplitude is $1\ \mu\text{V}$ and the response is a sinusoid of the same frequency with an amplitude of 5 V. What is the magnitude of the transfer function of the system at the frequency of the sinusoid, expressed in decibels (dB)?

Signal Ratio

$$20\log_{10}\left(\frac{5}{10^{-6}}\right) = 20\log_{10}\left(5\times 10^{6}\right) = 134\ \text{dB}$$

8. An ideal DT lowpass filter has a transfer function magnitude of 3 at $\Omega = 0$ and a radian cutoff frequency of $\pi/4$. Find the numerical value of the transfer function magnitude at

(a) $\Omega = \pi/2$ 0

All DT filters have periodic frequency responses. $\pi/2$ is greater than $\pi/4$ and less than $2\pi - \pi/4$ which is where the lower corner of the next periodic replica occurs.

(b) $\Omega = 7\pi/8$ 0

$7\pi/8$ is greater than $\pi/4$ and less than $2\pi - \pi/4$ which is where the lower corner of the next periodic replica occurs.

(c) $\Omega = 22\pi$ 3

If the filter's transfer function is 3 for radian frequencies $-\pi/4 < \Omega < \pi/4$ it is also 3 for radian frequencies $2n\pi - \pi/4 < \Omega < 2n\pi + \pi/4$, n an integer.