Student ID Number $\square\square\square - \square\square - \square\square\square\square$ (No name please)

Solution of ECE 316 Final Examination S06

1. The inverse Laplace transform of $\mathrm{H}(s) = \dfrac{s-1}{s^2(s+3)}$ can be expressed in the form $\mathrm{h}(t) = \left[At + B\left(1 - e^{-bt}\right)\right]\mathrm{u}(t)$. Find the numerical values of A, B and b.
A = <u>-1/3</u> , B = <u>4/9</u> , b = <u>3</u>

$$\mathrm{H}(s) = -\frac{1/3}{s^2} + \frac{4/9}{s} - \frac{4/9}{s+3} \Rightarrow \mathrm{h}(t) = -(1/3)\mathrm{ramp}(t) + (4/9)\left(1 - e^{-3t}\right)\mathrm{u}(t)$$

or

$$\mathrm{H}(s) = -\frac{1/3}{s^2} + \frac{4/9}{s} - \frac{4/9}{s+3} \Rightarrow \mathrm{h}(t) = \left[-t/3 + (4/9)\left(1 - e^{-3t}\right)\right]\mathrm{u}(t)$$

2. A signal that has no signal power for any frequency $|f| > f_m$ where f_m is finite is called a <u>bandlimited</u> signal.

3. The inverse z transform of $\mathrm{H}(z) = 6\dfrac{z(z+1/2)}{z^2+4/9}$ can be expressed in the form $\mathrm{h}[n] = A\alpha^n\left[\cos(\Omega_0 n) + B\sin(\Omega_0 n)\right]\mathrm{u}[n]$.
What are the numerical values of A, B, α and Ω_0?
A = <u>6</u> , B = <u>3/4</u> , α = <u>2/3</u> , Ω_0 = <u>$\pi/2$</u>

$$\mathrm{H}(z) = 6\frac{z^2 + z/2}{z^2 + 4/9} = 6\left[\frac{z^2 - z\alpha\cos(\Omega_0)}{z^2 + 2\alpha\cos(\Omega_0) + \alpha^2} + \frac{1}{2\alpha\sin(\Omega_0)}\frac{z\alpha\sin(\Omega_0)}{z^2 + 2\alpha\cos(\Omega_0) + \alpha^2}\right]$$

where $\alpha = 2/3$ and $\Omega_0 = \pi/2$.

$$\mathrm{H}(z) = 6\left[\frac{z^2}{z^2 + 4/9} + \frac{3}{4}\frac{2z/3}{z^2 + 4/9}\right]$$

$$\mathrm{h}[n] = 6(2/3)^n\left[\cos(\pi n/2) + (3/4)\sin(\pi n/2)\right]\mathrm{u}[n]$$

4. What must be true of the poles of a continuous-time-system transfer function for the system to be stable? (Only a <u>precise</u> answer will be accepted as correct.)

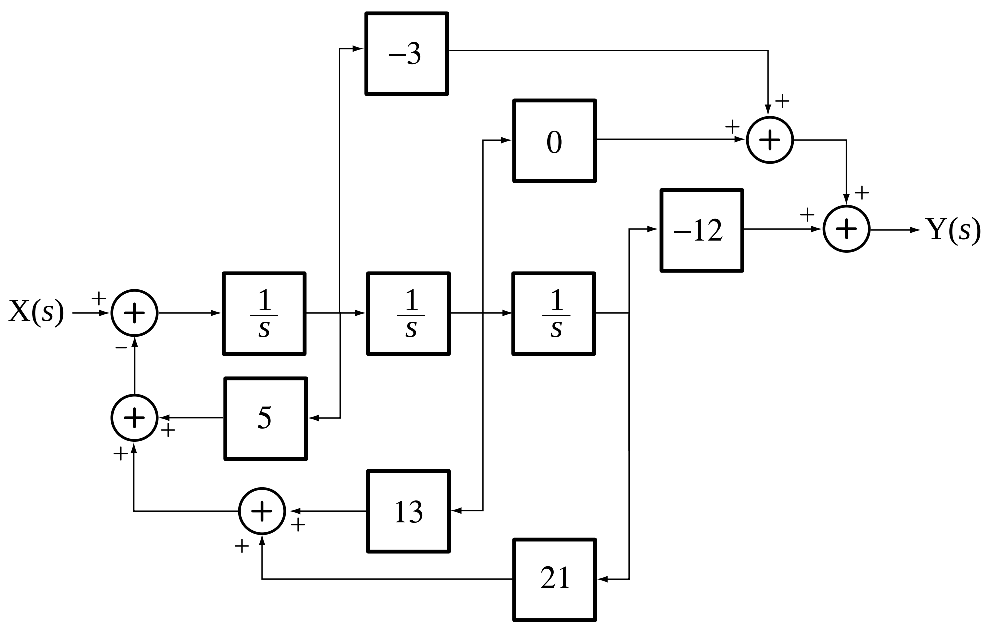
   They must all lie in the open left half-plane (not including the ω axis).

5. The loop transfer function of a feedback system has 4 finite poles and 1 finite zero. What is the angle (in radians) between the asymptotes of the system root locus?

   Angle = $\underline{2\pi / 3}$ radians

6. Fill in the numerical values in the gain blocks below for a system whose transfer function is $\mathrm{H}(s) = -3\dfrac{s^2+4}{(s+3)(s^2+2s+7)}$. (Every block should have a number, even if it is zero.)

$$\mathrm{H}(s) = \frac{-3s^2 - 12}{s^3 + 5s^2 + 13s + 21}$$



7. What relationship between the number of finite poles and the number of finite zeros of the loop transfer function of a discrete-time feedback system with an adjustable gain K in the forward path <u>always</u> <u>guarantees</u> that the system will be unstable at some finite value of K?

If the number of finite poles exceeds the number of finite zeros,the system will be unstable at some finite value of K. (Also if the number of zeros exceeds the number of poles the system will be unstable at some finite value of K but this describes a non-causal system and it is therefore unrealizable with real physical components.)

8. If the step response of a second-order system overshoots and rings before settling to its final value, the system is <u>underdamped</u>.

   Although this is the answer I was looking for I also accepted "stable" since it makes a correct sentence.

9. A signal that is bandlimited cannot be timelimited but a signal that is timelimited can be bandlimited.

   False

10. The Laplace transform can represent signals that cannot be represented by a Fourier transform. Give one example.

    $\mathrm{x}(t) = e^{\alpha t}\,\mathrm{u}(t) \ , \ \alpha > 0$

    The answer $e^{\alpha t}\,\mathrm{u}(t)$ is not adequate because for some values of α the Fourier transform does exist. Also the answer $e^{\alpha t} \ , \ \alpha > 0$ is incorrect because the Laplace transform cannot be found.

11. What must be true of the poles of a discrete-time system transfer function for the system to be stable? (Only a <u>precise</u> answer will be accepted as correct.)

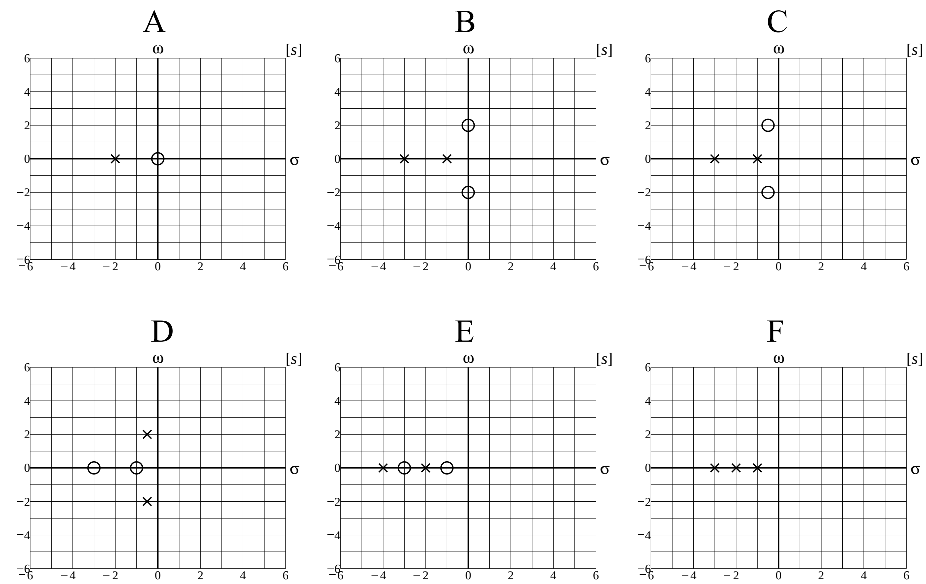
    <u>All the poles must lie inside the open unit circle (not on the unit circle).</u>

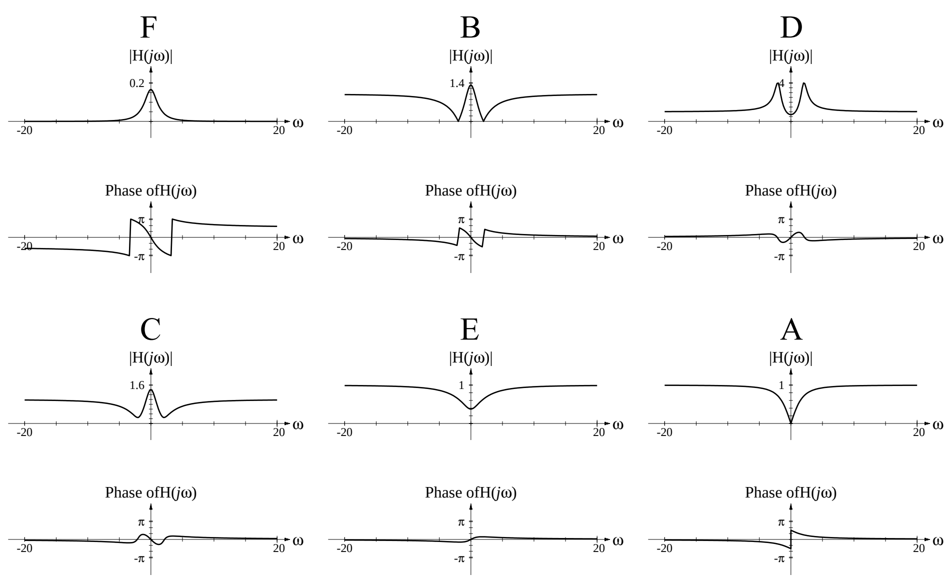
12. If $f_s = 200 = 1/T_s$ and $z = e^{sT_s}$ and a segment of the ω axis of the s plane maps into the complete unit circle of the z plane exactly one time, what is the numerical length of that segment of the ω axis?
    Length = $\underline{2\pi / T_s = 400\pi}$

13. What is the requirement on the number of points used in the FFT algorithm to make it most efficient?
    It should be a positive integer power of 2.

    The answer "a power of 2" is not adequate. Any real number is some power of 2 but not necessarily an integer power of 2.

14. Enter on the lines provided above the frequency response graphs below, the corresponding letter designation from the pole-zero graphs above them.





15. The frequency response of a discrete-time system is always a <u>periodic</u> function of F or Ω.

16. What is the principal difference between the DFT and the DTFS?
<u>A scaling constant which is the number of points</u>

17. For the ramp response of a unity-gain feedback system to have zero steady state error how many poles at $s=0$ must the forward path transfer function have?
Number of poles = <u>2 or more</u>

18. Let $\mathrm{x}_s[n]=\mathrm{x}(nT_s)$ where f_s is the sampling rate and $f_s=1/T_s$, and let $\mathrm{x}_s[n]\xleftrightarrow{\mathcal{F}}\mathrm{X}_s(j\Omega)$ and let $\mathrm{x}(t)\xleftrightarrow{\mathcal{F}}\mathrm{X}(f)$. Knowing f_s, it is always possible to exactly determine $\mathrm{X}_s(j\Omega)$ from $\mathrm{X}(f)$ but it is not always possible to exactly determine $\mathrm{X}(f)$ from $\mathrm{X}_s(j\Omega)$.

True

19. A discrete-time signal is formed by sampling a 20 kHz sinusoid at 50 kHz. What is the numerical value of the frequency of another sinusoid of the same amplitude that could yield exactly the same samples for all time if sampled at the same rate?
Frequency = <u>70 kHz, 120 kHz, ...</u>

20. The time-scaling property of the unilateral Laplace transform is $\mathrm{g}(at)\xleftrightarrow{\mathcal{L}}(1/a)\mathrm{G}(s/a)\ ,\ a>0$. Why is a required to be positive?

If it were negative $\mathrm{g}(at)$ would be non-causal and the relationship between the time and frequency domain functions would no longer be unique.

21. Give an example of a function $\mathrm{H}(s)$ for which the final-value theorem does not apply.

$$\mathrm{H}(s)=\frac{1}{s-\alpha},\ \ \mathrm{H}(s)=\frac{1}{s^2+\alpha}\ \ ,\ \alpha>0$$

22. Feedback can sometimes make a stable system unstable.

True

23. Every root locus begins on the <u>poles</u> of the loop transfer function and terminates on the <u>zeros</u> of the loop transfer function.

24. The ideal (but impractical) interpolating function to recreate a continuous-time signal from samples taken from it is the <u>sinc</u> function.

25. Feedback can sometimes make an unstable system stable.

True

26. A continuous-time filter transfer function has a zero at $s = 0$. Which of the digital filter design techniques below will guarantee that the system transfer function of the discrete-time (digital) filter will have a zero at $z = 1$? Circle all correct answers.

Impulse Invariant | $\boxed{\text{Finite Difference}}$

$\boxed{\text{Matched } z\text{-Transform}}$ | $\boxed{\text{Direct Substitution}}$ | $\boxed{\text{Bilinear}}$

Solution of ECE 316 Final Examination S06

1. The inverse Laplace transform of $\mathrm{H}(s)=\dfrac{s+1}{s^2(s+2)}$ can be expressed in the form $\mathrm{h}(t)=\left[At+B\left(1-e^{-bt}\right)\right]\mathrm{u}(t)$. Find the numerical values of A, B and b.

   A = <u>1/2</u> , B = <u>1/4</u> , b = <u>2</u>

$$\mathrm{H}(s)=\frac{1/2}{s^2}+\frac{1/4}{s}-\frac{1/4}{s+2}\Rightarrow \mathrm{h}(t)=(1/2)\mathrm{ramp}(t)+(1/4)\left(1-e^{-2t}\right)\mathrm{u}(t)$$

or

$$\mathrm{H}(s)=\frac{1/2}{s^2}+\frac{1/4}{s}-\frac{1/4}{s+3}\Rightarrow \mathrm{h}(t)=\left[t/2+(1/4)\left(1-e^{-2t}\right)\right]\mathrm{u}(t)$$

2. What relationship between the number of finite poles and the number of finite zeros of the loop transfer function of a discrete-time feedback system with an adjustable gain K in the forward path <u>always</u> <u>guarantees</u> that the system will be unstable at some finite value of K?

   If the number of finite poles exceeds the number of finite zeros,the system will be unstable at some finite value of K. (Also if the number of zeros exceeds the number of poles the system will be unstable at some finite value of K but this describes a non-causal system and it is therefore unrealizable with real physical components.)

3. What must be true of the poles of a continuous-time-system transfer function for the system to be stable? (Only a <u>precise</u> answer will be accepted as correct.)

   They must all lie in the open left half-plane (not including the ω axis).

4. The time-scaling property of the unilateral Laplace transform is $\mathrm{g}(at)\xleftrightarrow{\mathcal{L}}(1/a)\mathrm{G}(s/a)$, $a>0$. Why is a required to be positive?

   If it were negative $\mathrm{g}(at)$ would be non-causal and the relationship between the time and frequency domain functions would no longer be unique.

5. The inverse z transform of $\mathrm{H}(z) = 5\frac{z(z-1/2)}{z^2+1/9}$ can be expressed in the form $\mathrm{h}[n] = A\alpha^n\left[\cos(\Omega_0 n) + B\sin(\Omega_0 n)\right]\mathrm{u}[n]$.

What are the numerical values of A, B, α and Ω_0?

A = 5 , B = -3/2 , α = 1/3 , $\Omega_0 = \pi/2$

$$\mathrm{H}(z) = 5\frac{z(z-1/2)}{z^2+1/9} = 5\left[\frac{z^2 - z\alpha\cos(\Omega_0)}{z^2 + 2\alpha\cos(\Omega_0) + \alpha^2} - \frac{1}{2\alpha\sin(\Omega_0)}\frac{z\alpha\sin(\Omega_0)}{z^2 + 2\alpha\cos(\Omega_0) + \alpha^2}\right]$$

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$$\mathrm{H}(z) = 5\left[\frac{z^2}{z^2+1/9} - \frac{3}{2}\frac{z/3}{z^2+1/9}\right]$$

$$\mathrm{h}[n] = 5(1/3)^n\left[\cos(\pi n/2) - (3/2)\sin(\pi n/2)\right]\mathrm{u}[n]$$

6. Feedback can never make an unstable system stable.

False

7. A signal that is bandlimited cannot be timelimited and a signal that is timelimited cannot be bandlimited.

True

8. For the ramp response of a unity-gain feedback system to have zero steady state error how many poles at $s = 0$ must the forward path transfer function have?

Number of poles = 2 or more

9. If $f_s = 80 = 1/T_s$ and $z = e^{sT_s}$ and a segment of the ω axis of the s plane maps into the complete unit circle of the z plane exactly one time, what is the numerical length of that segment of the ω axis?

Length = $2\pi/T_s = 160\pi$

10. A signal that has no signal power for any frequency $|f| > f_m$ where f_m is finite is called a bandlimited signal.

11. The loop transfer function of a feedback system has 5 finite poles and 1 finite zero. What is the angle (in radians) between the asymptotes of the system root locus?

Angle = $\underline{\pi / 2}$ radians

12. What is the requirement on the number of points used in the FFT algorithm to make it most efficient?
It should be an integer power of 2.

The answer "a power of 2" is not adequate. Any real number is some power of 2 but not necessarily an integer power of 2.

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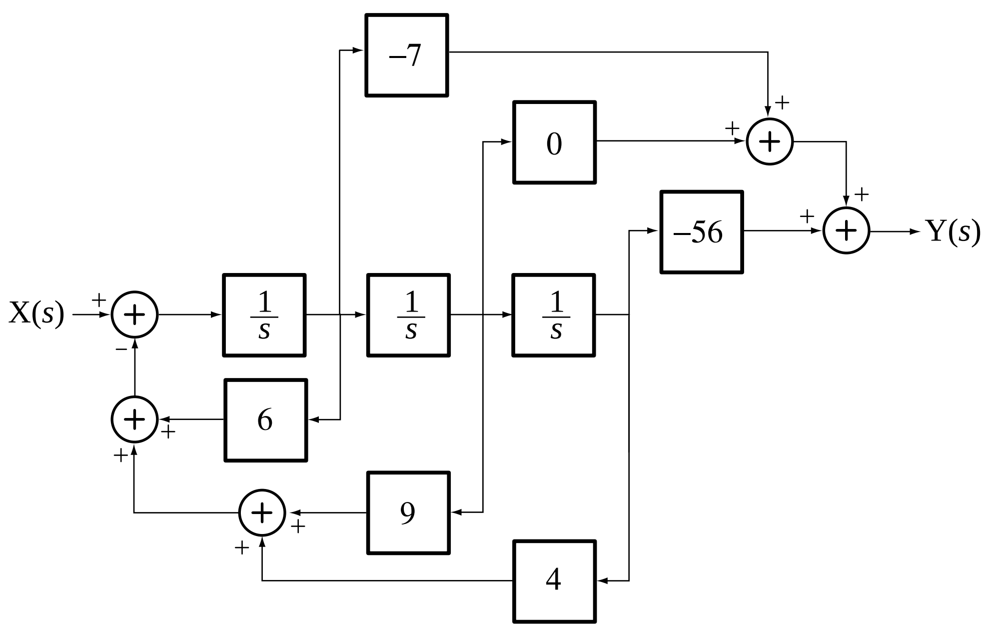
$$\mathrm{H}(s) = \frac{-7s^2 - 56}{s^3 + 6s^2 + 9s + 4}$$



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17. A discrete-time signal is formed by sampling a 10 kHz sinusoid at 50 kHz. What is the numerical value of the frequency of another sinusoid of the same amplitude that could yield exactly the same samples for all time if sampled at the same rate?
Frequency = <u>60 kHz, 110 kHz, ...</u>

18. Let $x_s[n] = x(nT_s)$ where f_s is the sampling rate and $f_s = 1/T_s$, and let $x_s[n] \xleftrightarrow{F} X_s(j\Omega)$ and let $x(t) \xleftrightarrow{F} X(f)$. Knowing f_s, it is always possible to exactly determine $X(f)$ from $X_s(j\Omega)$ but it is not always possible to exactly determine $X_s(j\Omega)$ from $X(f)$.

True

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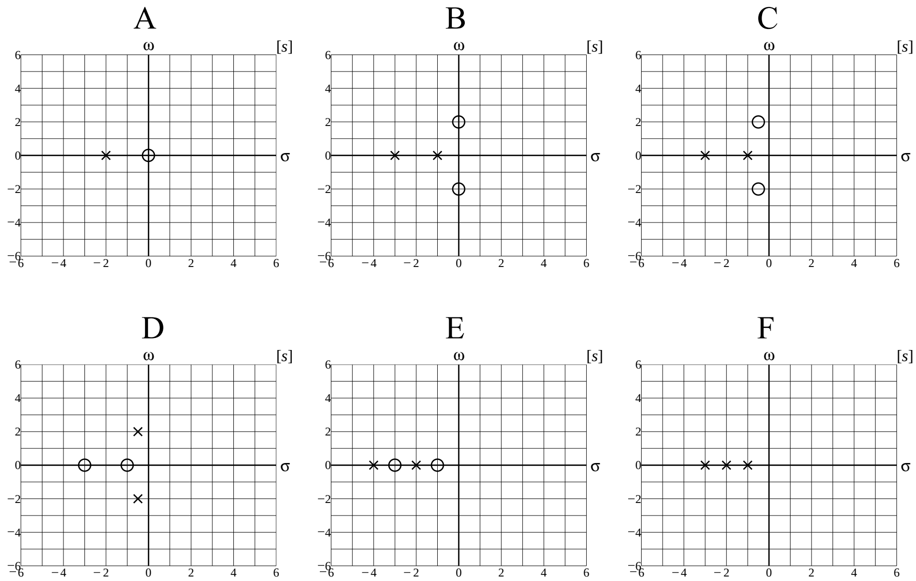

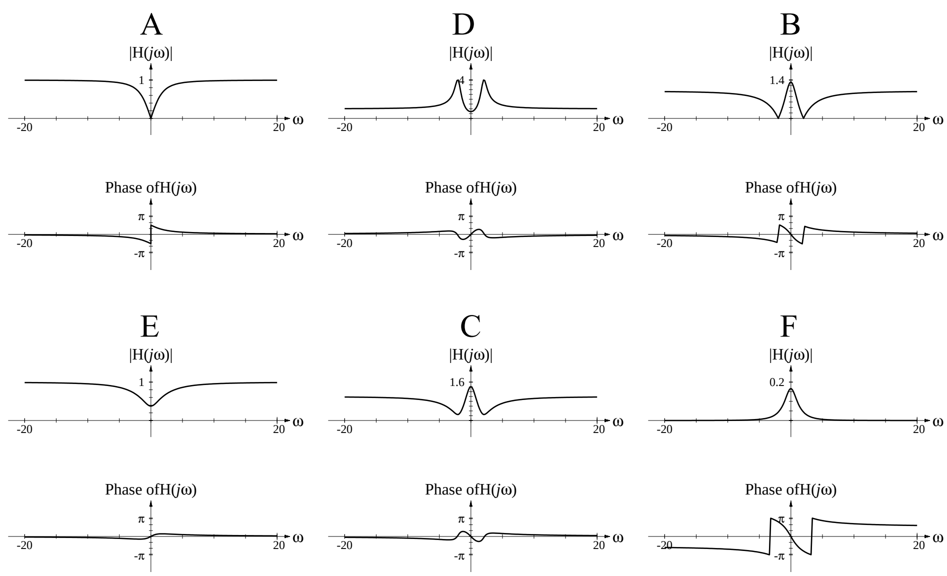

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